Math 208 - Quiz 7, March 12, 2009 Name:___________________Key______________________

Problem 1. Find the general solution on $\mathbb{R}$ to the differential equation $y^{(6)} - y = 0$.

Solution 1. The characteristic polynomial to this equation is $r^6 - 1$. To find the 6 roots of this polynomial we use Euler's identity $1 = e^{2k\pi i}$. We then have $r = 1^{1/6} = (e^{2k\pi i})^{1/6} = e^{\frac{k}{3}\pi i} = \cos(\frac{k}{3}\pi) + i\sin(\frac{k}{3}\pi)$. For $k = 0, \ldots, 5$ we then find the roots

$$r = 1, \quad \frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad -\frac{1}{2} + i\frac{\sqrt{3}}{2}, \quad -1, \quad -\frac{1}{2} - i\frac{\sqrt{3}}{2}, \quad \frac{1}{2} - i\frac{\sqrt{3}}{2}.$$

Now that we have the roots of the characteristic equation we know the formula for the general solution and so we have

$$y = c_1 e^t + c_2 e^{-t} + c_3 e^{\frac{t}{2}} \sin(\frac{\sqrt{3}}{2}t) + c_4 e^{\frac{t}{2}} \cos(\frac{\sqrt{3}}{2}t) + c_5 e^{-\frac{t}{2}} \sin(\frac{\sqrt{3}}{2}t) + c_6 e^{-\frac{t}{2}} \cos(\frac{\sqrt{3}}{2}t),$$

where $c_1, \ldots, c_6$ are constants.