Math 208 - Quiz 6, February 26, 2009 Name:__________Key__________

Problem 1. Find the general solution on $\mathbb{R}$ to the differential equation $y'' - 2y' + y = e^t$.

Solution 1. First of all note that the characteristic polynomial of the associated homogeneous equation $y''-2y'+y=0$ is $r^2 - 2r + 1 = (r-1)^2$. Hence the general solution of the homogeneous equation is $y_h = c_1e^t + c_2te^t$, where c_1 and c_2 and constants.

To find a particular solution to the non-homogeneous equation we use the method of undermined coefficients to conclude that a solution must be of the form t^sAe^t. When $s = 0, 1$ we see from above that we get solutions to the homogeneous equations and hence this will not work, thus we look at $s = 2$.

When $s = 2$ we have $y_p = At^2e^t$ hence $y_p' = At^2e^t + 2Ate^t$ and $y_p'' = At^2e^t + 4Ate^t + 2Ae^t$. Therefore $e^t = y_p'' - 2y_p + y_p = (A - 2A + A)t^2e^t + (4A - 4A)te^t + (2A)e^t = (2A)e^t$ and so $A = 1/2$. We then get the general solution for the non-homogeneous equation above as:

$$y = c_1e^t + c_2te^t + \frac{1}{2}t^2e^t.$$