Math 208 - Quiz 5, February 19, 2009 Name:__________________Key______________________

Problem 1. Find a solution on $\mathbb{R}$ to the differential equation $y'' + 2y' + 5y = 0$ subject to the initial conditions $y(0) = 1$, $y'(0) = 0$.

Solution 1. The characteristic polynomial associated to this equation is $r^2 + 2r + 5 = (r+1)^2 - (2i)^2 = (r+1+2i)(r+1-2i)$ hence we know that the general solution to this differential equation is $y = c_1 e^{-t} \sin 2t + c_2 e^{-t} \cos 2t$.

Hence $y' = -c_1 e^{-t} \sin 2t + 2c_1 e^{-t} \cos 2t - c_2 e^{-t} \cos 2t - 2c_2 e^{-t} \sin 2t$. Therefore the initial conditions give rise to the linear system of equaitons

$$c_2 = y(0) = 1,$$
$$2c_1 - c_2 = y'(0) = 0.$$

Solving this system we see that $c_2 = 1$ and $c_1 = 1/2$, hence our solution is

$$y = \frac{1}{2} e^{-t} \sin 2t + e^{-t} \cos 2t.$$