Math 208 - Quiz 4, February 5, 2009 Name:__________________Key______________________

Problem 1. Find the solution to the differential equation $(2y\sin x - e^x \sin y) + (-e^x \cos y - 2\cos x)y' = 0$.

Solution 1. This differential equation is of the form $M(x, y) + N(x, y)y' = 0$ where $M_y = 2\sin x - e^x \cos y = N_x$, hence it is exact. We therefore know that there exists some function $F(x, y)$ such that $F_x = M$ and $F_y = N$, furthermore we know that an implicit solution is of the form $F(x, y) = C$ where C is a constant. Thus we need only to find such an F.

Since $F_x = M$ we may integrate with respect to x to obtain

$$F = \int M dx + g(y) = -2y\cos x - e^x \sin y + g(y).$$

Then differentiating this with respect to y gives

$$-e^x \cos y - 2\cos x = N = F_y = -2\cos x - e^x \sin y + g'(y).$$

Hence $g'(y) = 0$ and so we may take $g = 0$ to obtain our implicit solution:

$$-2y\cos x - e^x \sin y = C,$$

where C is a constant.