Math 208 - Quiz 2, January 22, 2009 Name:__________Key__________

Problem 1. Find the general solution for the differential equation $ty' + (t+1)y = t$, for $t > 0$. Then compute $\lim_{t\to\infty} y(t)$.

Solution 1. Since $t > 0$ we may rewrite the above equation as $y' + \frac{t+1}{t}y = 1$, and note that this is a linear equation. We therefore multiply the equation by the integrating factor $\mu = \exp(\int \frac{t+1}{t}) = te^t$ to obtain $\frac{d}{dt}(yte^t) = te^t y' + te^t \frac{t+1}{t} y = te^t$. Integrating both sides with respect to t gives $yte^t = (t-1)e^t + C$ (use integration by parts). Solving for y gives the general solution

$$y = \frac{t-1}{t} + C\frac{e^{-t}}{t}.$$

Where C is a constant.

As $t \to \infty$ we see that $C\frac{e^{-t}}{t} \to 0$, and $\frac{t-1}{t} \to 1$, hence $\lim_{t\to\infty} y(t) = 1$.