Math 208 - Quiz 10, April 9, 2009 Name:__________________Key______________________

Problem 1. Use the Laplace transform to solve the initial value problem: $y'' - 4y' + 4y = 0$, $y(0) = 1$, $y'(0) = 1$.

Solution 1. By using the formula $\mathcal{L}\{f^{(n)}(t)\} = s^n \mathcal{L}\{f(t)\} - \Sigma_{k=1}^{n} s^{n-k} f^{(k-1)}(0)$ we may take the Laplace transform of the above equation to obtain

$$s^2\mathcal{L}\{y\} - s - 1 - 4s\mathcal{L}\{y\} + 4 + 4\mathcal{L}\{y\} = 0.$$

Solving for $\mathcal{L}\{y\}$ we get

$$\mathcal{L}\{y\} = \frac{s-3}{(s-2)^2} = \frac{1}{s-2} - \frac{1}{(s-2)^2}.$$

We know from class that the inverse Laplace transform of $\frac{1}{s-2}$ is e^{2t}, and the inverse Laplace transform of $\frac{1}{(s-2)^2}$ is te^{2t} hence by applying the inverse Laplace transform we can solve for y:

$$y = e^{2t} - te^{2t}.$$