Math 208 - Quiz 1, January 15, 2009 Name:__________Key__________

Problem 1. Solve each of the following initial value problems and graph the solution.

(a). $\frac{dy}{dt} = -y + 3$, $y(0) = 2$.

(b). $\frac{dy}{dt} = -2y + 1$, $y(0) = 1$.

(c). $\frac{dy}{dt} = y - 2$, $y(0) = 2$.

Solution 1. I'll show how to solve the above problems and leave the graphing as an exercise.

(a). Assuming that $y \neq 3$ we can rewrite the above equation as $\frac{1}{y-3}\frac{dy}{dt} = -1$. Integrating both sides we then have $\ln|y-3| = -t + C$. Then exponentiating we have $|y-3| = C_0 e^{-t}$.

At this point we may let C_0 be a positive or negative constant and hence we may drop the absolute value and solve for y to get $y = 3 + C_0 e^{-t}$. From the initial condition $y(0) = 2$ we see that $C_0 = -3 + y(0) = -1$ and so our solution is:

$$y = 3 - e^{-t}.$$

(b). Just as above if $y \neq 1/2$ we may rewrite the equation as $\frac{1}{y-1/2}\frac{dy}{dt} = -2$. Integrating we get $\ln|y-1/2| = -2t + c$. Exponentiating and solving for y gives $y = \frac{1}{2} + C_0 e^{-2t}$ and the initial condition $y(0) = 1$ gives us $C_0 = y(0) - \frac{1}{2} = \frac{1}{2}$. So that our solution is:

$$y = \frac{1}{2}(1 + e^{-2t}).$$

(c). Here $y(0) = 2$ and so we cannot divide by $y - 2$, however by looking at a slope field we see that $y = 2$ is actually a singular solution and so we are done, i.e. our solution is:

$$y = 2.$$