Math 208 - Exam 3, April 14, 2009 Name:______________________________

Problem 1 (20 points). Find the general solution to the differential equation $y' + xy = 1 + x$ in powers of x. You may leave your answer in the form of a recurrence relation for the coefficients of y, in any case however you should explicitly compute the first four terms of the power series expansion.

Problem 2 (20 points). Consider the differential equation $x^2y''-2xy'+cy=0$ where c is a real number. Find all values of c such that there exists a solution y of this differential equation which satisfies $\lim_{x\to\infty} y(x) = \infty$.

Problem 3 (20 points). Find a non-zero solution in powers of x to the differential equation $x^2y'' + 3xy' + (1 + x)y = 0$. Note that you do not have to find the general solution, any non-zero solution is fine.

Problem 4 (20 points). Find a solution to the initial value problem $y'' + y = g(t)$, $y(0) = 0$, $y'(0) = 1$, where

$$g(t) = \begin{cases} t/2, & 0 \le t < 6 \\ 3, & t \ge 6 \end{cases}$$

Problem 5 (20 points). Consider the integral equation $y(t) + \int_0^t (t-w)y(w)dw = 1$. Use the Laplace transform to find a solution y to this equation.

Hint: Use the convolution formula.

Problem 6 (Extra Credit - 10 points). Given $\delta > 0$, define the function $g_\delta : [0, \infty) \to \mathbb{R}$ by

$$g_\delta(t) = \begin{cases} 1/\delta, & 0 \leq t < \delta \\ 0, & t \geq \delta \end{cases}$$

Show that if $f : [0, \infty) \to \mathbb{R}$ is a continuous function then $\lim_{\delta \to 0}(f * g_\delta)(t) = f(t)$, for all $t > 0$.