Math 208 - Exam 1, February 10, 2009 Name:----------

Problem 1 (15 points). Find an explicit solution to the differential equation: $y' - (\tan t)y = \frac{1}{\cos t}$ for $\frac{-\pi}{2} < t < \frac{\pi}{2}$, subject to the initial condition $y(0) = -1$.

Hint: It may be useful to remember the formula: $(\ln(\cos t))' = -\tan t$, for $\frac{-\pi}{2} < t < \frac{\pi}{2}$.

Problem 2 (15 points). Suppose that a sum S is invested at the annual rate of return 8% compounded continuously.

(a). Find the time T required for the initial amount to double.

(b). How much should we invest today if we wanted to have the sum of $\$40,000$ in 20 years?

Problem 3 (20 points). A tank initially contains 100 gallons of water with 50 pounds of salt. Pure water is pumped into the tank at the rate of 2 gallons per minute. The mixture is then pumped at the same rate into a second tank which initially contains 100 gallons of pure water. The mixture is then allowed to flow out of the second tank again at the rate of 2 gallons per minute.

(a). Find an expression for the amount of salt in the first tank at time t.

(b). Find the time $t_{\max}$ when the second tank contains the most amount of salt. How much salt does the second tank contain at this time?

Problem 4 (15 points). Find an explicit solution for $x > \frac{1}{2}\sqrt{2}$ to the differential equation: $y' = \frac{4x^2+3y^2}{2xy}$, subject to the initial condition $y(1) = -2$.

Problem 5 (20 points). Find an implicit solution to the differential equation: $y' = -\frac{2y+3xy^2+1}{x+2x^2y}$.

Problem 6 (15 points). Find an explicit solution on $\mathbb{R}$ to the differential equation: $y'' + y' - 6y = 0$, subject to the initial conditions $y(0) = 1$, $y'(0) = -1$.

Problem 7 (Extra Credit - 10 points). Find an explicit solution to the second order differential equation: $y'' = e^y y'$ for $t < 1$, subject to the initial conditions $y(0) = 0$, $y'(0) = 1$.

Hint: Use the substitution $v = \frac{dy}{dt}$ and solve for v in terms of y.