1. Find an equation for a plane which contains the line given by \((x, y, z) = t(1, 2, 0)\) and which forms an angle of \(\pi/3\) with the \(xy\)-plane.

We need to first find a normal vector \(n = (x_0, y_0, z_0)\) for this plane. Note that since any vector parallel to a normal vector is again a normal vector, we may assume that \(n\) has length 1. We know that \(n\) must form an angle of \(\pi/3\) with \(k\), the normal vector of the \(xy\)-plane, this gives rise to the equation \(z_0 = n \cdot k = |n||k|\cos(\pi/3) = 1/2\).

Since the vector \((1, 2, 0)\) is parallel to the above given line, which is contained in the plane, we also know that \(n\) must be perpendicular to \((1, 2, 0)\). This gives rise to the equation \(x_0 + 2y_0 = (1, 2, 0) \cdot n = 0\).

Hence \(n\) is of the form \((-2y_0, y_0, 1/2)\), and since this must have length 1, we have \(4y_0^2 + y_0^2 + (1/4) = 1\), hence \(y_0 = \pm \sqrt{3}/20\). Note that there are two possibilities for \(y_0\) since there are two planes which satisfy the above conditions.

Since the plane contains the origin \(0\), after simplifying, we may find an equation for the plane given by 
\[
-2\sqrt{3}x + \sqrt{3}y + \sqrt{5}z = 0,
\]
or 
\[
2\sqrt{3}x - \sqrt{3}y + \sqrt{5}z = 0.
\]

2. Find parametric equations for the line of intersection of the planes \(x + y - z = 1\) and \(3x + 2y - z = 0\). Also find the angle between these two planes.

To find a point on this line we can for instance set \(z = 0\) and then use the above equations to solve for \(x\) and \(y\). In this case we get \(x = -2\) and \(y = 3\) so \((-2, 3, 0)\) is a point on the line. Also the direction of the line lives in both planes and so in particular is perpendicular to both normal vectors, therefore a vector which is parallel to the line is given by \((1, 1, -1) \times (3, 2, -1) = (1, -2, -1)\).

Thus an equation of the line is given by the vector equation 
\[(x, y, z) = (-2, 3, 0) + t(1, -2, -1),\]
or the parametric equations 
\[x = -2 + t, \quad y = 3 - 2t, \quad z = -t,\]
or the symmetric equations 
\[x + 2 = \frac{y - 3}{-2} = -z.\]

Then angle between these planes is given by 
\[
\cos \theta = \frac{|(1, 1, -1)(-1)(3, 2, -1)|}{|1(1, 1, -1)||3(3, 2, -1)|} = 6/\sqrt{42},
\]
and so \(\theta = \cos^{-1}(6/\sqrt{42})\).

3. Find the distance between the parallel planes \(2x - 2y - 6z = 1\) and \(x - y - 3z = -2\).
To find the distance between the planes we may take a point on the first plane (how about $(0, 0, -\frac{1}{6})$) and find the distance from this point to the second plane. We can do this by the formula we derived in class so that the distance is

$$\frac{|0 + 0 + 3/6 + 2|}{\sqrt{1 + 1 + 9}} = \frac{5}{2\sqrt{11}}.$$ 

4. Find the cross-sections of the surface $2x^2 + 2y^2 + z^2 = 1$ in the planes $x = k$, $y = k$ and $z = k$. Sketch the surface.

When $x$ is a constant $k$ the cross-section is given by $2y^2 + z^2 = 1 - 2k^2$ which is the equation of an ellipse. Similarly when $y$, or $z$ is constant we get another ellipse. And so our equation describes an ellipsoid which I leave to you to sketch.

5. Find the cross-sections of the surface $-x^2 - 2y^2 + 3z^2 = 1$ in the planes $x = k$, $y = k$ and $z = k$. Sketch the surface.

When $x$ is a constant $k$ the cross-section is given by $-2y^2 + 3z^2 = 1 + k^2$ which is a hyperbola, note that $k^2 > 0$ and so for no value of $k$ will we obtain the asymptotes of this hyperbola. Similarly when $y$ is a constant we also get a hyperbola. When $z$ is a constant $k$ we get $-x^2 + 2y^2 = 1 - 3k^2$ which describes an ellipse.

Thus our surface is a hyperboloid in the direction of the $z$-axis, which has two sheets. I leave it to you to sketch this.

6. Show that the curve of intersection of the surfaces $x^2 + 2y^2 - z^2 + 3x = 1$ and $2x^2 + 4y^2 - 2z^2 - 5y = 0$ lies in a plane.

If we take the second equation, and subtract from it twice the first equation we obtain

$$-5y - 6x = -2.$$ 

This we recognize as the equation of a plane, and since any point on the intersection of the above surfaces must satisfy this equation also, we have shown that the intersection of the surfaces lies on this plane.

7. A surface consists of all the points $P$ such that the distance from $P$ to the plane $z = 1$ is the same as the distance from $P$ to the point $(0, 1, -2)$. Find an equation for, and sketch this surface.

Given a point $P = (x, y, z)$, the distance from $P$ to the plane $z = 1$ is $|1 - z|$. Also, the distance from $P$ to the point $(0, 1, -2)$ is $\sqrt{x^2 + (y - 1)^2 + (z + 2)^2}$. If these distances are equal then we have the equation

$$(1 - z)^2 = x^2 + (y - 1)^2 + (z + 2)^2.$$ 

Simplifying this equation gives

$$x^2 + (y - 1)^2 + (6z + 3) = 0,$$

which we recognize as the equation of an elliptic paraboloid. I leave this to you to sketch.