HOMEWORK 3, MATH 175 - FALL 2010

DUE FRIDAY SEPTEMBER 24TH

This homework assignment covers Sections 14.1 - 14.3 in the book.

1. Find $\lim_{t\to 0} r(t)$ where $r : \mathbb{R} \to \mathbb{R}^3$ is the vector function given by $r(t) = (e^{-t^2}, \frac{t^2}{\sin^2 t}, \cos(t-\pi))$.

2. Find a vector function that represents the curve of intersection of the two surfaces given by $x^2 + 4y^2 = 1$ and $z = xy$.

3. Consider the vector functions $r(t) = (|t|, \sin t, -\sqrt{|t|})$ and $s(t) = (t^2, \sin t, 1 - \cos t)$. Determine where the functions r, s, and $f(t) = r(t) \cdot s(t)$ are differentiable, and compute the derivatives.

4. Find parametric equations for the tangent line to the curve given by $x = 1 - t$, $y = \sqrt{t}$, $z = e^t - t$ at the point $(0, 1, e - 1)$.

5. Let $r(t) = (2t + t, 3e^t - t, e^t + 3t - 7)$ find $f(t) = r(t) \cdot (r'(t) \times r''(t))$. (Hint: First find $f'(t)$ and $f(0)$ and then solve a differential equation.)

6. Let $r(t) = (2e^{-t}\cos(3e^{-t^2}), 2e^{-t}\sin(3e^{-t^2}), 2\sqrt{1 - e^{-2t}})$, where $t \geq 0$, compute $r(t) \cdot r'(t)$.

7. Find the length of the curve $r(t) = (\ln\cos t, \sin t, \cos t)$, for $0 \leq t \leq \pi/4$.

8. Find the unit tangent vector, the unit normal vector, and the curvature at time t of the curve given by $r(t) = (\sin t, e^t, t^2)$.

9. Suppose that $r : \mathbb{R} \to \mathbb{R}^3$ denotes a smooth curve which is parameterized with respect to arc length. Show that $r'(t)$ and $r''(t)$ are always perpendicular. Find an example where this is false if r is not parameterized with respect to arc length.