HOMEWORK 2, MATH 175 - FALL 2009

This homework assignment covers Sections 13.5 - 13.7 in the book.

1. Find an equation of a plane which contains the point $(1, -1, 2)$ and is parallel to the plane $2x - y + z = 1$.

A normal vector of the plane given by the equation $2x - y + z = 1$ is $(2, -1, 1)$. Hence since we are looking for a parallel plane it should have the same normal vector. Since $(1, -1, 2)$ is a point on the plane we know that the equation of the plane is then given by

$$(x, y, z) \cdot (2, -1, 1) = (1, -1, 2) \cdot (2, -1, 1) = 5,$$

or

$$\boxed{2x - y + z = 5.}$$

2. Find parametric equations for the line of intersection of the planes $x - y + z = 1$ and $3x - 2y - z = 0$. Also find the angle between these two planes.

To find a point on this line we can for instance set $z = 0$ and then use the above equations to solve for x and y. In this case we get $x = -2$ and $y = -3$ so $(-2, -3, 0)$ is a point on the line. Also the direction of the line lives in both planes and so in particular is perpendicular to both normal vectors, therefore a vector which is parallel to the line is given by $(1, -1, 1) \times (3, -2, -1) = (3, 4, 1)$.

Thus an equation of the line is given by the vector equation

$$(x, y, z) = (-2, -3, 0) + t(3, 4, 1),$$

or the parametric equations

$$\boxed{x = -2 + 3t, \quad y = -3 + 4t, \quad z = t,}$$

or the symmetric equations

$$\frac{x + 2}{3} = \frac{y + 3}{4} = z.$$

3. Find the distance between the parallel planes $2x + 4y - 6z = 1$ and $x + 2y - 3z = -2$.

Note that these planes have parallel normal vectors and hence are indeed parallel.

To find the distance between the planes we may take a point on the first plane (how about $(0, 0, \frac{-1}{6})$ and find the distance from this point to the second plane. We can do this by the formula we derived in class so that the distance is

$$\frac{|0 + 0 + 3/6 + 2|}{\sqrt{1 + 4 + 9}} = \boxed{\frac{5}{2\sqrt{14}}.}$$

4. Find the cross-sections of the surface $x^2 + y^2 + 2z^2 = 1$ in the planes $x = k$, $y = k$ and $z = k$. Sketch the surface.

When x is a constant k the cross-section is given by $y^2 + 2z^2 = 1 - k^2$ which is the equation of an ellipse. Similarly when y, or z is constant we get another ellipse. And so our equation describes an ellipsoid which I leave to you to sketch.

5. Find the cross-sections of the surface $-x^2 - 2y^2 + 3z^2 = 1$ in the planes $x = k$, $y = k$ and $z = k$. Sketch the surface.

When x is a constant k the cross-section is given by $-2y^2 + 3z^2 = 1 + k^2$ which is a hyperbola, note that $k^2 > 0$ and so for no value of k will we obtain the asymptotes of this hyperbola. Similarly when y is a constant we also get a hyperbola. When z is a constant k we get $-x^2 + 2y^2 = 1 - 3k^2$ which describes an ellipse.

Thus our surface is a hyperboloid in the direction of the z-axis, which has two sheets. I leave it to you to sketch this.

6. Change $(2, -3, 1)$ from rectangular to cylindrical and also spherical coordinates.

For the cylindrical coordinates we have $\tan\theta = y/x = -3/2$ hence $\theta = \tan^{-1}(-3/2)$ also $r = \sqrt{x^2 + y^2} = \sqrt{13}$. Therefore we have

$$\boxed{\text{Cylindrical}: \quad (\sqrt{13}, \tan^{-1}(-3/2), 1).}$$

For the Spherical coordinates θ is the same as above, however $\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{14}$ and $\cos\phi = z/\rho = 1/\sqrt{14}$ hence we have

$$\boxed{\text{Spherical}: \quad (\sqrt{14}, \tan^{-1}(-3/2), \cos^{-1}(1/\sqrt{14})).}$$

7. Write the equation $x^2 + y^2 + z^2 = 2x$ in cylindrical and also spherical coordinates.

For cylindrical we have $x^2 + y^2 = r^2$ and $x = r\cos\theta$ hence we may rewrite the equation as

$$\boxed{r^2 + z^2 = 2r\cos\theta.}$$

For the spherical case we have $x^2 + y^2 + z^2 = \rho^2$ and $x = \rho\sin\phi\cos\theta$ hence when we plug this in and divide by ρ we have

$$\boxed{\rho = 2\sin\phi\cos\theta.}$$

8. The parabola $z = 9y^2$, $x = 0$ is rotated about the z-axis. Write an equation of the resulting surface in cylindrical coordinates.

If we look at the cross-section of the resulting surfaces when z is a constant k. then since we are rotating around the z axis we will get a circle centered at the point $(0, 0, k)$. Moreover the circle will go through the point $(0, \sqrt{k}/3, k)$ since it has to satisfy the formula $z = 9y^2$ when $x = 0$. Thus the circle will have radius $\sqrt{k}/3$. The formula for this circle in polar coordinates is then given by $r^2 = k/9$.

For an arbitrary z we then obtain the surface as $r^2 = z/9$ or $\boxed{z = 9r^2.}$