

HOMEWORK 7, MATH 175 - FALL 2009

DUE FRIDAY NOVEMBER 6TH (AT THE BEGINNING OF CLASS)

This homework assignment covers Sections 16.3 and 16.4 in the book.

1. Find the volume of the solid which is under the surface $z = 2x + y^2$ and above the region bounded by $x = y^2$ and $x = y^3$.

2. Evaluate the integral $\int_0^{\sqrt{\pi}} \int_y^{\sqrt{\pi}} \cos(x^2) dx dy$.

3. Evaluate the integral $\iint_D \sin(x - y)e^{(x-y)^2} dA$ where D is a disk of radius 2 whose center is $(1, 1)$.

4. Evaluate the integral $\iint_R \sqrt{4 - x^2 - y^2} dA$, where R is the region that lies above the x -axis within the circle $x^2 + y^2 = 4$.

5. Find the area of the region enclosed by the curve $r = 4 + 3 \cos \theta$.

6. Given a non-negative function $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ we define the improper integral

$$\begin{aligned} \iint_{\mathbb{R}^2} f(x, y) dA &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x, y) dx dy \\ &= \lim_{a \rightarrow \infty} \iint_{D_a} f(x, y) dA = \lim_{b \rightarrow \infty} \iint_{R_b} f(x, y) dA. \end{aligned}$$

Where D_a is the disk of radius a and center $(0, 0)$ and R_b is the square with vertices $(\pm b, \pm b)$.

Use this to compute the integral $\int_{-\infty}^{\infty} e^{-x^2} dx$. (Hint: $(\int_{-\infty}^{\infty} e^{-x^2} dx)^2 = (\int_{-\infty}^{\infty} e^{-x^2} dx)(\int_{-\infty}^{\infty} e^{-y^2} dy) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} e^{-(x^2+y^2)} dx dy$.)

7. Compute the integral $\int_{-\infty}^{\infty} x^2 e^{-x^2} dx$.