

HOMEWORK 4, MATH 175 - FALL 2009

DUE FRIDAY OCTOBER 9TH (AT THE BEGINNING OF CLASS)

This homework assignment covers Sections 15.1 - 15.3 in the book.

1. Find and sketch the domain of the function  $f(x, y) = \sqrt{x-y} + \ln(y^2)$ . What is the range of this function?
2. Sketch the contour map and the graph of the function  $f(x, y) = 4x^2 + y^2 + 1$ .
3. Sketch the domain and contour map of the function  $f(x, y) = \ln(y/x)$ .
4. Find the limit and prove it exists or prove that it does not exist for:
  - (a).  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 \sin^2 y}{x^2 + 2y^2}$ .
  - (b).  $\lim_{(x,y) \rightarrow (0,0)} \frac{x \sin y}{x^2 + 2y^2}$ .
  - (c).  $\lim_{(x,y) \rightarrow (1,1)} \frac{e^x \ln y}{x^2 + 2y^2}$ .
  - (d).  $\lim_{(x,y,z) \rightarrow (0,0,0)} \frac{x^2 y - z^2}{x^2 + y^2 + z^2}$ .
5. Let  $f(x, y) = \begin{cases} 0 & \text{if } y \leq 0 \text{ or } y \geq x^4 \\ 1 & \text{if } 0 < y < x^4. \end{cases}$ 
  - (a). Show that  $f(x, y) \rightarrow 0$  as  $(x, y) \rightarrow (0, 0)$  along any path through  $(0, 0)$  of the form  $y = mx^a$  with  $a < 4$ .
  - (b). Show that  $f$  is discontinuous at  $(0, 0)$ .
6. Find the partial derivatives of the following functions:
  - (a).  $f(x, y) = y^4 - 3xy - e^x$ .
  - (b).  $f(x, y) = \tan xy$ .
  - (c).  $f(x, y, z) = z^2 e^{xy^2 z}$ .
  - (d).  $f(x_1, x_2, \dots, x_n) = \sqrt{x_1^2 + x_2^2 + \dots + x_n^2}$ .
7. Find all second partial derivatives of the function  $f(x, y) = \sin^2(x - y)$ .