

Math 175 - Exam 2, October 28, 2009

Name:-----

*Problem 1* (20 points). Consider the function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  given by  $f(x, y) = \begin{cases} \frac{x^2}{2x^2+y^2} & \text{if } (x, y) \neq (0, 0) \\ 0 & \text{if } (x, y) = (0, 0) \end{cases}$

Draw the contour map (level curves) for  $f$  and find where the function is continuous.

*Problem 2* (20 points). Find an equation for the tangent plane to the surface given by the equation  $z = xe^{x(y-1)}$  at the point  $(1, 1, 1)$ .

*Problem 3* (20 points). Consider the function  $g(x, y, z) = x^2 + y^2 + z^2 - xyz$ , where  $x, y$ , and  $z$  are all functions of  $s$ , and  $t$  which satisfy the equations  $x = s$ ,  $y = t$ , and  $s^3 + t^3 + z^3 + 6stz = 0$ . Find  $\partial g / \partial t$  when  $s = 1$  and  $t = 0$ .

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*Problem 4* (20 points). Let  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  be the function given by  $f(x, y) = x^3 + y^3 - 3xy$ .

(a). Find all local minimums, maximums, and saddle points of  $f$ .

(b). Find the absolute minimum and maximum of  $f$  when restricted to the domain  $D = \{(x, y) \mid 0 \leq y \leq x \leq 4\}$  i.e.,  $D$  is the region above the  $x$ -axis, to the left of the line  $x = 4$  and under the line  $x = y$ .

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*Problem 5* (20 points). Find the volume of the solid which is bounded by the surface  $z = xye^{x^2y}$ , and the planes  $x = 0$ ,  $x = 1$ ,  $y = 1$ ,  $y = 2$ , and  $z = 0$ .

*Problem 6* (Extra Credit - 10 points, no partial credit). Compute  $\int_0^1 \int_0^1 f(x, y) dx dy$  where  $f(x, y) = \frac{1}{x+y}$ . (Hint: You may want to use the fact that  $f(x, y) = f(y, x)$ .)