

Math 175 - Exam 1, September 30, 2009

Name:-----

*Problem 1* (20 points). Let three points be given by  $P = (2, 1, 5)$ ,  $Q = (-1, 3, 4)$ , and  $R = (3, 0, 6)$ .

(a). Find an equation of the plane which contains  $P$ ,  $Q$ , and  $R$ .

(b). Find the area of the triangle  $PQR$ .

*Problem 2* (20 points). Consider the plane  $P$  given by the equation  $x - y + 3z = 1$ , and the line  $L$  given by  $(x, y, z) = (1, 1, 0) + t(3, -1, -1)$ ,  $t \in \mathbb{R}$ .

(a). Find the angle between the plane  $P$  and the line  $L$ .

(b). Find an equation of a parallel plane  $P'$  such that the line segment given by the part of  $L$  which lies between the two planes  $P$  and  $P'$  has length  $3\sqrt{11}$ .

*Problem 3* (20 points). Classify and sketch the surface given by the equation  $4y^2 + z^2 - x - 16y - 4z + 20 = 0$ .

*Problem 4* (20 points).

- (a). Write the equations  $z = \sqrt{x^2 + y^2}$  and  $x^2 + y^2 + z^2 = z$  in spherical coordinates.
- (b). Describe in words and sketch the two surfaces given by the equations.

*Problem 5* (20 points). Let  $r : [0, \infty) \rightarrow \mathbb{R}^3$  be given as  $r(t) = (\frac{4}{3}t^{3/2}, 2t, \frac{1}{2}t^2)$ .

(a). Find the length of the resulting curve from the point  $(0, 0, 0)$  to  $(\frac{32}{3}, 8, 8)$ .

(b). Find the curvature of the resulting curve at the point  $(\frac{4}{3}, 2, \frac{1}{2})$ .

*Problem 6* (Extra Credit - 10 points, no partial credit). Suppose that  $r : \mathbb{R} \rightarrow \mathbb{R}^3$  defines a smooth curve which is parameterized with respect to arc length. Show that  $r'(t)$  and  $r''(t)$  are always perpendicular. Find an example where this is false if  $r$  is not parameterized with respect to arc length.