Problem 1. Find the general solution (on $\mathbb{R}$) to the following differential equation:

$$y^{(4)} + y^{(2)} = 0.$$

Solution 1. The associated polynomial to the above equation is $P(x) = x^4 - x^2$, which factors as $P(x) = (x^2 + 1)x^2 = (x - i)(x + i)x^2$. Since the root at 0 has order 2 we know that 1 and x are both solutions to the differential equation. Also since there are complex roots at i and $-i$ we know that $\cos x = (e^{ix} - e^{-ix})/2$ and $\sin x = (e^{ix} + e^{-ix})/2i$ are both solutions.

Hence we have 4 linearly independent solutions and therefore the general solution is given as a linear combination:

$$y = c_1 + c_2 x + c_3 \cos x + c_4 \sin x,$$

where c_1, c_2, c_3, and c_4 are constants.