Math 196 - Quiz 7, October 30, 2008 Name:____

Problem 1. Compute the determinant of the following matrix in order to see if it is invertible:

$$A = \begin{pmatrix} 4 & 7 & 10 & 2 \\ 1 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 \\ 1 & 7 & -1 & 2 \end{pmatrix}.$$

Solution 1. If we switch two rows or columns of the matrix A then it changes the sign of the determinant and so we have:

$$\det A = -\det \begin{pmatrix} 0 & 2 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 4 & 7 & 10 & 2 \\ 1 & 7 & -1 & 2 \end{pmatrix} = \det \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 7 & 4 & 10 & 2 \\ 7 & 1 & -1 & 2 \end{pmatrix}.$$

We may then compute the above determinant directly so that:

$$\det A = \det \begin{pmatrix} 2 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 7 & 4 & 10 & 2 \\ 7 & 1 & -1 & 2 \end{pmatrix}$$

$$= 2\det \begin{pmatrix} 1 & 2 & 0 \\ 4 & 10 & 2 \\ 1 & -1 & 2 \end{pmatrix} = 2(\det \begin{pmatrix} 10 & 2 \\ -1 & 2 \end{pmatrix} - 2\det \begin{pmatrix} 4 & 2 \\ 1 & 2 \end{pmatrix})$$

$$= 2((20+2) - 2(8-2)) = 20.$$

Since this is non-zero we know that the matrix A is invertible.