Math 196 - Quiz 6, October 9, 2008 Name:________________________________

Problem 1. Let V be the subspace in $\mathbb{R}^4$ given by all solutions to the following homogeneous system of linear equations:

$$2x_1 + 4x_2 - 6x_3 = 0$$
$$x_1 + x_2 - x_3 + 3x_4 = 0$$
$$x_1 - x_2 + 3x_3 + 9x_4 = 0.$$

1. Find a linearly independent set for V.
2. Find a spanning set for V.
3. Find a basis for V.
4. What is the dimension of V?

Solution 1. Note that a basis is both linearly independent and spanning thus our answer for part 3 will also work for parts 1 and 2.

To find a basis lets solve the system of linear equations:

$$\begin{pmatrix} 2 & 4 & -6 & 0 & 0 \\ 1 & 1 & -1 & 3 & 0 \\ 1 & -1 & 3 & 9 & 0 \end{pmatrix} \overset{\frac{1}{2}\text{R1}}{\longrightarrow} \begin{pmatrix} 1 & 2 & -3 & 0 & 0 \\ 1 & 1 & -1 & 3 & 0 \\ 1 & -1 & 3 & 9 & 0 \end{pmatrix} \overset{\text{R2} - \text{R1}}{\longrightarrow} \begin{pmatrix} 1 & 2 & -3 & 0 & 0 \\ 0 & -1 & 2 & 3 & 0 \\ 1 & -1 & 3 & 9 & 0 \end{pmatrix}$$

$$\overset{\text{R3} - \text{R1}}{\longrightarrow} \begin{pmatrix} 1 & 2 & -3 & 0 & 0 \\ 0 & -1 & 2 & 3 & 0 \\ 0 & -3 & 6 & 9 & 0 \end{pmatrix} \overset{\text{R3} - 3\text{R2}}{\longrightarrow} \begin{pmatrix} 1 & 2 & -3 & 0 & 0 \\ 0 & -1 & 2 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \overset{-\text{R2}}{\longrightarrow} \begin{pmatrix} 1 & 2 & -3 & 0 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\overset{\text{R1} - 2\text{R2}}{\longrightarrow} \begin{pmatrix} 1 & 0 & 1 & 6 & 0 \\ 0 & 1 & -2 & -3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Hence $V = \left\{ \begin{pmatrix} -s - 6t \\ 2t + 3s \\ s \\ t \end{pmatrix} | s, t \in \mathbb{R} \right\} = \text{span} \left\{ \begin{pmatrix} -1 \\ 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -6 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\}.$

Thus our solution for parts 1, 2, and 3 is given by $\left\{ \begin{pmatrix} -1 \\ 3 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -6 \\ 2 \\ 0 \\ 1 \end{pmatrix} \right\}.$

The dimension is just the number of vectors in a basis and so in this case it is 2.