Math 196 - Quiz 3, September 18, 2008 Name:________Key________

Problem 1. Using the substitution $v = \ln y$, find a solution to the differential equation $\frac{dy}{dx} + 2y\ln y = 2y(x^2 + x)$ given the initial condition $y(1) = e$.

Solution 1. If we substitute $v = \ln y$ (we will assume y is positive at this point) then we have $y = e^v$, and hence by the chain rule $\frac{dy}{dx} = e^v \frac{dv}{dx}$. Substituting this into the differential equation above we get the new differential equation

$$e^v \frac{dv}{dx} + 2ve^v = 2e^v(x^2 + x),$$

dividing by e^v gives us

$$\frac{dv}{dx} + 2v = 2(x^2 + x),$$

which is a linear equation.

We solve the linear equation by multiplying both sides of the equation by e^{2x} and applying the Leibniz rule to get

$$\frac{d}{dx}(e^{2x}v) = 2(x^2 + x)e^{2x}.$$

Integrating the above equation gives us

$$e^{2x}v = x^2e^{2x} + C,$$

where C is a constant. From the initial condition $y(1) = e$ we see that $v(1) = \ln(y(1)) = \ln e = 1$ and so $C = 0$, hence dividing by e^{2x} gives us

$$v = x^2.$$

Substituting back $y = e^v$ we see that

$$y = e^{x^2}.$$