Math 196 - Quiz 10, December 2, 2008 Name:__________________________________

Problem 1. Find a particular solution on $\mathbb{R}$ to the differential equation:

$$y'' + 2y' + 3y = 4xe^x.$$

Solution 1. Using the method of undetermined coefficients we first notice that neither $4xe^x$ nor it's derivative $4xe^x + 4e^x$ are solutions to the homogeneous differential equation $y'' + 2y' + 3y = 0$. Hence we check to see if a linear combination of the above functions gives a solution, i.e. we look at a function of the form $y_p(x) = Axe^x + Be^x$, where A and B are constants. Note that xe^x and e^x are linearly independent functions.

Taking derivatives we then have $y_p' = Axe^x + (A + B)e^x$, and $y_p'' = Axe^x + (2A + B)e^x$. Hence

$$4xe^x = y_p'' + 2y_p' + 2y_p$$

$$= (Axe^x + (2A + B)e^x) + 2(Axe^x + (A + B)e^x) + 3(Axe^x + Be^x)$$

$$= 6Axe^x + (4A + 6B)e^x.$$

Hence $6A = 4$, and $4A + 6B = 0$, i.e. $A = \frac{2}{3}$, and $B = -\frac{2}{3}A = -\frac{4}{9}$. We then have that

$$y_p = \frac{2}{3}xe^x - \frac{4}{9}e^x$$

is a solution to our differential equation.

Note that since the roots of $r^2 + 2r + 3$ are $r = -1 \pm \sqrt{2}i$, by solving the associated homogeneous equation we can also find the general solution to the above differential equation as

$$y = c_1e^{-x}\sin(\sqrt{2}x) + c_2e^{-x}\cos(\sqrt{2}x) + \frac{2}{3}xe^x - \frac{4}{9}e^x,$$

where c_1 and c_2 are constants.