

Problem 1. In each of the following cases determine whether or not the subset W is a subspace of the vector space V .

- (1) W is the set of all vectors in $V = \mathbb{R}^3$ such that $x_1 + x_2 = 0$.
- (2) W is the set of all vectors in $V = \mathbb{R}^3$ such that $x_1 + x_2 = 1$.
- (3) W is the set of all vectors in $V = \mathbb{R}^3$ such that $x_1x_2 = 0$.
- (4) $W = \{0\} \subset V = \mathbb{R}^4$.
- (5) W is the set of all vectors of the form $s \begin{pmatrix} 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ where $s, t \in \mathbb{R}$ and $V = \mathbb{R}^2$.
- (6) W is the set of all vectors of the form $s \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ or of the form $t \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ where $s, t \in \mathbb{R}$ and $V = \mathbb{R}^2$.
- (7) W is the set of all vectors v in $V = \mathbb{R}^2$ such that $\begin{pmatrix} -1 & 2 \\ 3 & 2 \end{pmatrix} v = 0$.
- (8) W is the set of all matrices A in $V = M_{2 \times 2}(\mathbb{R})$ such that $A \begin{pmatrix} 1 \\ 4 \end{pmatrix} = 0$.
- (9) W is the set of all polynomials p such that $p(2) = 0$, where V is the vector space of all polynomials.
- (10) W is the set of all polynomials p such that $p(0) = 2$, where V is the vector space of all polynomials.