Homework 4, Section 3.3, Problem 32.

Problem 3.3.32.

Show that the 2×2 matrix

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

is row equivalent to the 2×2 identity matrix provided that $ad - bc \neq 0$.

Solution. If $ad - bc \neq 0$ then we must have that at least one of a or c is not equal to 0. If $a = 0$ then $c \neq 0$ and hence we can switch the two rows to obtain a new row equivalent matrix where the first position is not 0. Hence we may assume that $a \neq 0$.

Now we may divide the first row by a so that that we get the row equivalent matrix:

$$\begin{pmatrix} 1 & b/a \\ c & d \end{pmatrix}$$

Then we may subtract c times the first row from the second row in order to obtain:

$$\begin{pmatrix} 1 & b/a \\ 0 & d - cb/a \end{pmatrix}$$

Since $ad - bc \neq 0$ we also have that $d - bc/a \neq 0$ and hence we may divide the second row by this in order to obtain:

$$\begin{pmatrix} 1 & b/a \\ 0 & 1 \end{pmatrix}$$

Finally by multiplying the second row by b/a and subtracting it from the first we obtain the identity matrix:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

Note that although the problem did not ask us to show this, it is also true that the converse holds. That is if

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

is row equivalent to the 2×2 identity matrix then we must have $ad - bc \neq 0$.