

Homework 1, Section 1.4, Problem 26.

Problem 1.4.26. Find an explicit particular solution to the initial value problem:

$$\frac{dy}{dx} = 2xy^2 + 3x^2y^2, y(1) = -1$$

Solution. By separating the variables we obtain the new equation

$$\frac{1}{y^2} \frac{dy}{dx} = 2x + 3x^2,$$

when $y \neq 0$. Integrating both sides of the equation with respect to x we obtain

$$\frac{-1}{y} = \int \frac{1}{y^2} dy = \int \frac{1}{y^2} \frac{dy}{dx} dx = \int (2x + 3x^2) dx = x^2 + x^3 + C.$$

Substituting in the initial condition $y(1) = -1$ we see that $1 = \frac{-1}{-1} = 1^2 + 1^3 + C = C + 2$, hence $C = -1$ and so solving the above equation for y we find the explicit solution:

$$y = \frac{-1}{x^2 + x^3 - 1}.$$