 Name:____

Problem 1 (20 points). In each of the following cases, matrices A, and B are given. Determine if there exists a matrix S such that $SA = B$. If so then find such an S, and if not then give a reason why not.

1. $A = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$.

2. $A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & 0 & -1 \\ 1 & 0 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 2 & 0 & -1 \\ 1 & 0 & 3 \\ 1 & 1 & 1 \end{pmatrix}$.

3. $A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 2 & -1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 2 & 1 \\ 2 & -1 & 0 \\ 2 & -1 & 0 \end{pmatrix}$.

Problem 2 (10 points). Find a basis for each of the eigenspaces of the matrix $A = \begin{pmatrix} 2 & 0 & -1 & 1 \\ 0 & 2 & 1 & 3 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

Problem 3 (20 points). For each of the cases below determine if the matrix A is diagonalizable with real coefficients. (You do not need to actually find a P such that $P^{-1}AP$ is diagonal, but you do need to give a reason why such a P does or does not exist.)

1. $A = \begin{pmatrix} \sqrt{2} & \pi \\ \pi & \sqrt{2} \end{pmatrix}$

2. $A = \begin{pmatrix} 3 & 2 & -1 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{pmatrix}$

3. $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$

4. $A = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 0 & 2 & 3 & 4 & 5 & 6 \\ 0 & 0 & 3 & 4 & 5 & 6 \\ 0 & 0 & 0 & 4 & 5 & 6 \\ 0 & 0 & 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 6 \end{pmatrix}$

Problem 4 (15 points). Find the solution to the homogeneous linear differential equation $y'' - 2y' + 2y = 0$ which satisfies the initial conditions $y(0) = 1, y'(0) = 0$.

Problem 5 (20 points). Consider the homogeneous linear differential equaiton $y^{(3)} - xy'' + 9y' - 9xy = 0$ together with solutions $f_1(x) = \sin 3x$, $f_2(x) = \cos 3x$, $f_3(x) = 3\sin x - 4\sin^3 x$. Determine whether or not f_1, f_2, and f_3 are linearly independent. Show your work.

Problem 6 (15 points). Find the general solution to linear differential equation

$$y^{(3)} + 3y'' + 3y' + y = 2\cos x - 2\sin x$$

knowing that a particular solution is given by $y_0(x) = \sin x$.

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