ABSTRACTS OF TALKS

(the information in brackets refers to session numbers)

On Simultaneous Approximation in Function Spaces [C-5B]

E. Abu-Sirhan
Tafila Technical University, Tafila, Jordan
esarhan@ttu.edu.jo

The problem of simultaneous approximation in function spaces has attracted many researcher recently. Major results on the space of vector valued continuos functions started to appear early nineties. In 2002, results on simultaneous approximation in p-Bochner integrable functions were published. The object of this paper is to give a charactarization for the subspaces of Bochner integrable functions space to be simultaneously proximinal.

Lipschitz Constant for Vector Valued Approximation [C-5B]

Mohammad A. AlQudah* and James R. Angelos
Central Michigan University, Mount Pleasant, MI, USA
alqud1ma@cmich.edu

We present a formula for the local Lipschitz constant for uniform approximation of f on a discrete subset X of $[-1,1]$ from a generalized Haar subspace of dimension n, under the restriction that X has exactly $(m+1)$ points, where m is the dimension of the component spaces comprising the generalized Haar subspace G.

Numerical Methods for Fully Nonlinear Equations [M-13B]

Gerard Awanou
Northern Illinois University
awanou@math.niu.edu

While the theory of second order fully nonlinear equations has received considerable attention, there is a paucity of numerical methods, especially finite element methods, for these equations. As it is not in general possible to weaken the order of the equations through integration by parts, spaces of C^1 spline functions form an appropriate framework to approximate the solution of these equations. We introduce a general framework for numerical solution of these equations and illustrate the performance of the approach with numerical experiments using the spline element method. We treat the examples of the Monge-Ampère and Pucci equations and discuss the convergence of our algorithms.

Sharp Inequalities of Kolmogorov Type for Hypersingular Integrals and Some Applications [M-1B]

Vladislav Babenko
Dnepropetrovsk National University, Ukraine
babenko.vladislav@gmail.com

In this talk we present several new sharp inequalities of Kolmogorov type for hypersingular integrals in univariate and multivariate case.In particular, we present new inequalities for fractional degrees of differential operators of elliptic and parabolic type. In addition, we present some applications of the obtained results.

On the L_p-error of Adaptive Interpolation by Splines on Box Partitions [M-6A]

Y. Babenko*, T. Leskevich, J.-M. Mirebeau
Sam Houston State University, TX, USA
babenko@shsu.edu

We will show that

$$\lim_{N\to\infty} N^{\frac{n}{2}} \|f - s(f,N)\|_p = \frac{M_p}{n!} \left\| \sqrt{\frac{\partial^n f}{\partial x^n} \frac{\partial^n f}{\partial y^n}} \right\|_{L_{\frac{2p}{np+2}}([0,1]^2)}$$

by splines $s(f,N)$ that are constructed on adaptively generated box partition and when restricted to each element of the partition are polynomials of total degree n and of degree $n-1$ in each variable.

We will show that

$$\lim_{N\to\infty} N^{\frac{n}{2}} \|f - s(f,N)\|_p = \frac{M_p}{n!} \left\| \sqrt{\frac{\partial^n f}{\partial x^n} \frac{\partial^n f}{\partial y^n}} \right\|_{L_{\frac{2p}{np+2}}([0,1]^2)}$$

and provide the description of the exact constant M_p and suggest some interpolation schemes.

Finally, we will provide a more general approach and the solution of a problem which incorporates this and other schemes.

Sampling and Recovery of Multidimensional Bandlimited Functions [C-16A]

Benjamin Bailey
Texas A&M University, College Station, TX, USA
abailey@math.tamu.edu

In this talk, we investigate frames for $L_2[-\pi,\pi]^d$ consisting of exponential functions in connection to oversampling and nonuniform sampling of bandlimited functions. We analyze a multidimensional nonuniform oversampling formula for bandlimited functions with a general frequency domain along with a computationally manageable version of this formula. High dimensional approximation of biorthogonal functions is also considered.

Reduction of Planar Orthogonality to Non-Hermitian Orthogonality on Contours [M-6B]

F. Balogh*, M. Bertola, S. Y. Lee, K. McLaughlin
Concordia University, Montreal, Quebec, Canada
fbalogh@mathstat.concordia.ca

Motivated by normal random matrix models of mathematical physics, the asymptotics of orthogonal polynomials in the complex plane with respect to weights of the form $\exp(-|z|^2 + h(z))$ is studied, where $h(z)$ is perturbation function dictated by the model in question.

For certain special cases the standard two-dimensional orthogonality relations in the complex plane are shown to be equivalent to a set of non-hermitian orthogonality relations with respect to a varying analytic weight function on a combination of certain complex contours. This step simplifies the asymptotic analysis considerably: in many interesting cases one may use the powerful Riemann-Hilbert method to obtain strong asymptotics for the corresponding orthogonal polynomials. The talk is devoted to a special case studied in detail in a recent joint work with M. Bertola, S.Y. Lee and K. McLaughlin.

Minimal Energy Spherical Splines on Clough-Tocher Triangulations for Hermite Interpolation [C-10A]

Victoria Baramidze
Western Illinois University, Macomb, Illinois
V-Baramidze@wiu.edu

Using a minimal energy approach we construct cubic spherical splines interpolating function values and its first order derivatives over the Clough-Tocher partition of an arbitrary spherical triangulation. The use of the macro-element allows us to reduce the size of the linear system involved in the algorithm. We obtain bounds for the approximation error of a given sufficiently smooth function by the minimal energy Hermite interpolating spline. In addition, we prove that the minimizers of energy functionals with different homogeneous extensions are equivalent in the sense that they all converge to the sampled function, and the order of convergence is independent of the extension. We conclude with some numerical examples.

Weighted Uniform Rational Approximation to Schur Functions [M-8B]

L. Baratchart*, S. Kupin, V. Lunot, and M. Olivi
INRIA, Universite de Provence, INRIA, INRIA, France
baratcha@sophia.inria.fr

Let f be a Schur function in the unit disk D and μ the positive measure on the unit circle T whose Herglotz transform is $(1+zf)/(1-zf)$. Let α_k a sequence in D which is not hyperbolically separated, and E the set of accumulation points on T. If μ' is smooth and strictly positive on a neighbourhood of E, and moreover if the support of μ_s is disjoint from E, we prove that the Wall rational functions A_n/B_n interpolating f at α_j for $1 \leq j \leq n$ converge to f on T in the uniform norm weighted by the Poisson kernel at α_n.

Divided Differences and Exponential Brownian Motion [talk canceled]

Brad Baxter* and Raymond Brummelhuis
Birkbeck College, University of London, London, England
b.baxter@bbk.ac.uk

We provide a surprising new application of classical approximation theory to a fundamental asset-pricing model of mathematical finance. Specifically, we calculate an analytic value for the correlation coefficient between exponential Brownian motion and its time average, and we find the use of divided differences greatly elucidates calculations, providing an easy path to several new results. As an application, we find that this correlation coefficient is always at least $1/\sqrt{2}$, in so doing deriving a seemingly new Turan inequality. Further, using the Hermite–Genocchi integral relation, we demonstrate that all moments of the time average are certain divided differences of the exponential function. We also prove that these moments agree with the somewhat more complex formulas obtained in the stochastic literature.

Cluster Assumption and Sparsity in the Eigenfunction Basis [M-3A]

Kaushik Sinha and Mikhail Belkin*
Ohio State University
mikhail.belkin@gmail.com

The cluster assumption in semi-supervised learning states natural groups or ”clusters” in the data should have same or similar labels. I will discuss how this geometric assumption is related to the sparsity of the classification function in a certain basis associated to a convolution operator. I will also show how such sparse basis subsets can be found and provide some comparisons to the standard sparse learning methods.

Diffusive Wavelets on Groups and Homogenous Spaces [C-18B]

S. Bernstein*, S. Ebert, J. Wirht
Freiberg Univ. of Mining and Technology, FRG, Imperial College London, UK
swanhild.bernstein@math.tu-freiberg.de

The construction of wavelets relies on translations and dilations which are perfectly given in R. On other manifolds, the sphere for example, it is not obvious what translations and dilations are. One way out is to use the translation given with the manifold and to incooperate the dilation in to the kernel of a semigroup, i.e. the semigroup parameter is the dilation parameter. Because the archetype of a semigroup is the diffusion semigroup we will call these wavelets diffusive wavelets.

The concept of diffusive wavelets can be explained in the language of representation theory of Lie groups and within the framework of the group Fourier transform. The practical tools for the construction are zonal functions and approximate identities. We will explain the construction by using examples. The simplest situation appears for the unit circle which can be generalized to tori in higher dimensions. Next, we consider the sphere, which we represent as the homogenous space $SO(n+1)/SO(n)$. A reconstruction formula is obtained because all kernels lead to approximate identities. We will use the Abel-Poisson as well as the Gauß-Weierstraß kernel.

To construct wavelets that are invariant under finite group reflections we use approximate identities in the Dunkl setting in order to construct spherical Dunkl wavelets, which do not involve the knowledge of the interwining operator, the Dunkl translation or the Dunkl transform. We demonstrate the practicallity of the approach with Abel-Poisson-Dunkl wavelets.

Adaptive Approximation of Surfaces [M-3B]

Peter Binev* and Nira Dyn
University of South Carolina and Tel Aviv University
binev@math.sc.edu

We propose adaptive multiscale renement algorithms for approximating and encoding surfaces by piecewise linear triangular meshes. We establish rates of approximation of these algorithms in the Hausdor metric using certain characteristics of the approximated surface.

Phase Transitions for Sparse Approximation Algorithms [M-1A]

Jeffrey D. Blanchard*, Coralia Cartis, Jared Tanner, and Andrew Thompson
Grinnell College, Iowa, USA and University of Edinburgh, UK
blanchaj@math.grinnell.edu

Several algorithms have been shown to exactly recover sparse vectors provided the RIP constants of the measurement matrix satisfy a certain bound. By establishing bounds on the RIP constants we investigate when a Gaussian measurement matrix will satisfy, with high probability, the sufficient conditions on these algorithms for exact sparse recovery. Following the phase transition framework advocated by Donoho, this provides a method for direct, quantitative comparison of sufficient conditions for sparse approximation algorithms.

Optimal Recovery of Functions and Integrals on Classes Defined by a Majorant for the Modulus of Continuity [M-1B]

V. F. Babenko, B. D. Bojanov, S. V. Borodachov*
Dnepropetrovsk University, University of Sofia, Towson University
sborodachov@towson.edu

On the class of functions defined on a compact region $D \subset \mathbf{R}^d$, $d \in \mathbf{N}$, with a given majorant for their moduli of continuity, we study the problem about optimal global recovery of functions and their integrals along D. We assume that the functions are known on a proper compact subset $G \subset D$, which belongs to a certain class of subsets of D of the same type (such as the class of n-point sets in D, the class of all intersections of D with unions of n hyperplanes, or the class of unions of n embedded hyperspheres, etc). The region D is assumed to be a ball, a parallelepiped, or a convex centrally symmetric body. For a number of cases, we find or describe the most "informative" sets G from the above mentioned classes and the algorithms, for global recovery of functions or their integrals on D with the minimal worst case error. In every case considered, a linear algorithm is found among the optimal ones.

Optimal Nonlinear Approximations and Errors Reduction for Numerical Reuleaux Method (NRM) Pseudo-Rigid Bodies Dynamics [C-18B]

Francisco Casesnoves
SIAM-SIAG Research Group in Geometric Design, UK
casesnoves2@engineering.com

The Classical Reuleaux Method (CRM) is frequently used in Engineering to determine the exact Instantaneous Rotation Center (IRC) of a rigid body in arbitrary movement. However, if the body in movement is pseudo-rigid, the CRM has to be approximated and modified numerically to conform the shape and density distribution changes of this pseudo-rigid body, into the Numerical Reuleaux Method (NRM). The applications of this Nonlinear Numerical Algorithm in Engineering and Bioengineering are useful for Engineering Machines or Devices and, for instance, also in Artificial Lumbar Implants. The pseudo-rigid body whose shape changes, under Mechanical forces or human Biomechanical loads during the machine or human movements, complicates the IRC determination. But in such cases, the IRC can be optimally approximated by using the NRM. The synthesis of the theoretical basis of this method is shown, and the principal Theorems and Propositions, are presented. A group of Objective Functions for Numerical Optimization of NRM is detailed, with Nonlinear and Statistical Approximations. Further Nonlinear Statistical Approximations have been carried out beyond the studies developed by Bryant (1984), and Panjabi (1979-1981) for rigid bodies. Their approximations are related to a Mechanical Engineering Imaging Data Smoothing Method, to find the optimal IRC for rigid bodies. This 2D kinematic measurement smoothing data model has been statistically improved, and 2D and 3D approximations have been carried out to enhance the accuracy of its initial model for IRC determination. It also constitutes a Mechanical Engineering General Method, not only related to Biomechanical Orthopedic Research, but also for Mechanical Devices. The improvements of the Statistical and Geometrical Approximations of Bryant that have been made, could be extended to 3D in several standard-deviation related directions. The high-error and geometrical indetermination problem, when the CRM is used for small angles rotations, is approached with a new Differential Algorithm defined in a resulting 2D Nonlinear Formula. Simulations and Computational Techniques have been carried out with appropriate software to demonstrate the feasibility, existence and uniqueness of the optimal solution. The initial results, both for formulas and numerical simulations, agree to the Theoretical Calculations, and the IRC computation for 2 voxels shows to be simple and easy. A Theoretical development of this algorithm for large pseudo-rigid bodies, by using Approximation Monte Carlo Techniques, was already presented in 2007 SIAM Conference of Geometric Design and Computing (San Antonio, TX, USA).

Acknowledgements: This research contribution is dedicated with all my love and sincere gratitude to my parents. They have given me the Life and my Education.

Nonconvex Compressive Sensing and Dvoretzky's Theorem for Quasi-Normed Spaces [M-1A]

Rick Chartrand
Los Alamos National Laboratory, Los Alamos, New Mexico, USA
rickc@lanl.gov

In this talk, we relate results concerning the ability of nonconvex optimization to recover sparse signals from extremely limited data, to generalizations of geometric properties of Banach spaces to the quasi-normed setting.

The recently-emerged field of compressive sensing has produced powerful results related to the reconstruction of sparse images or signals from very few data. The combinatorial problem of finding the sparsest solution to a linear system can, in many circumstances, be solved efficiently using convex optimization. However, by using nonconvex objectives instead, signals can be reconstructed from even fewer data. The resulting optimization problems have many local minima, but efficient algorithms have been very successful at converging to the local minimum.

The theoretical justification of the fact that ℓ^p minimization with $p < 1$ outperforms ℓ^1 minimization is related to geometric properties of the space ℓ^p. In particular, Dvoretzky's Theorem, originally proved for Banach spaces but since extended to quasi-normed spaces, demonstrates that the unit ball ℓ^p can be sliced by large-dimensional subspaces to obtain a section that is almost spherical. This theorem and the success of nonconvex compressive sensing are ultimately consequences of concentration of measure properties. This and other connections will be drawn in this talk.

Generalization of Polynomial Interpolation at Chebyshev Nodes [C-10B]

Debao Chen* and Ward Cheney
OSU – Tulsa and UT Austin
debao.chen@okstate.edu

Previously, we generalized the Lagrange polynomial interpolation at Chebyshev nodes and studied the Lagrange polynomial interpolation at a special class of sets of nodes. This special class includes some well-known sets of nodes, such as zeros of the Chebyshev polynomials of first and second kinds, Chebyshev extrema, and equidistant nodes. In this paper, we view our previous work from a different perspective and further generalize and study the Lagrange polynomial interpolation at a larger class of sets of nodes. In particular, the set of optimal nodes is included in this extended class.

On the Use of T-splines in Isogeometric Analysis [M-8A]

Annalisa Buffa, Durkbin Cho*, Giancarlo Sangalli
IMATI - CNR, Pavia, Italy
durkbin@imati.cnr.it

NURBS-based isogeometric analysis has been applied in fields as diverse as fluid dynamics, structure mechanics and electromagnetics. A significant disadvantage of NURBS models is that they may have a large number of superfluous control points at each step of refinement. In contrast, T-splines allow the local refinement and retain precise geometric description as well, which is a main advantage in the isogeometric concept. However, theoretical issues concerning T-splines are open. In this talk, we present our recent results in mathematical understanding of T-splines techniques as a tool for local refinement. Questions related to linear independence and approximation properties will be addressed.

[1] A. Buffa, D. Cho, and G. Sangalli. Linear independence of the T-spline blending functions associated with some particular T-meshes. *Comput. Methods Appl. Mech. Engrg.*, (2009) DOI: 10.1016/j.cma.2009.12.004.
[2] T.W. Sederberg, D.L. Cardon, G.T. Finnigan, N.S. North, J. Zheng, and T. Lyche. T-spline simplication and local refinement. *ACM Transactions on Graphics*, 23(3):276–283, 2004.

High Dimensional Sparse Approximation of Stochastic-Parametric PDE's [P-2]

Albert Cohen
Université Pierre et Marie Curie, Paris
cohen@ann.jussieu.fr

Various mathematical problems are challenged by the fact they involve functions of a very large number of variables. Such problems arise naturally in learning theory, partial differential equations or numerical models depending on parametric or stochastic variables. They typically result in numerical difficulties due to the so-called "curse of dimensionality". We shall explain how these difficulties may be handled in the context of stochastic-parametric PDE's based on two important concepts: (i) variable reduction and (ii) sparse approximation.

Parameterizing Volumes and Creating Trivariate Splines for Geometric Modeling and Isogeometric Analysis [M-11A]

Elaine Cohen
School of Computing, University of Utah, Salt Lake City, UT 84112
cohen@cs.utah.edu

Isogeometric Analysis (IA) has been proposed as a methodology for bridging the gap between Computer Aided Design (CAD) and Finite Element Analysis (FEA). In order to support design and full 3D IA, new ab initio design methods must create suitable representations and approximation techniques must include parameterization methodologies for volumes. This presentation discusses some of the challenges in moving from current representations and datafitting technques towards this goal and demonstrates initial results.

Quasi-interpolation Methods for Multivariate Splines [P-14]

Oleg Davydov
University of Strathclyde, Glasgow, UK
oleg.davydov@strath.ac.uk

Given a set of locally supported basis functions ("B-splines") $B_1, \dots, B_n$, a linear quasi-interpolation operator is given by $Qf = \sum_{i=1}^{n} \lambda_i(f) B_i$, where λ_i are appropriate locally supported linear functionals. Quasi-interpolants Qf are a standard tool in theoretical estimates of the approximation power of splines. Their practical applications often require replacing λ_i by (in general non-linear) functionals $\tilde{\lambda}_i(f) = \lambda_i(p_i(f))$ where $p_i(f)$ is a local approximation of f. In this talk, we give an overview of recent results on quasi-interpolation in two and more variables, and related scattered data fitting methods.

On Anisotropic Hardy Spaces [M-6A]

S. Dekel*, P. Petrushev, and T. Weissblat
GE Healthcare, University of South Carolina, Tel-Aviv University
shai.dekel@ge.com

We generalize the spaces investigated in [1],[4], by constructing highly anisotropic Hardy spaces over the ellipsoid covers introduced in [2],[3]. Essentially, these spaces are equivalent if only if the quasi-distances induced on R^n by the ellipsoid covers are equivalent. We also generalize classic elements of the theory in this setting such as the various maximal functions, atomic decompositions, dual BMO spaces, etc. This is work in progress.

[1] M. Bownik, Anisotropic Hardy spaces and wavelets, *Mem. Amer. Math. Soc.* 164 (2003), no 781.
[2] W. Dahmen, S. Dekel and P. Petrushev, Two-level-split decomposition of anisotropic Besov spaces, *Constr. Approx.*, to appear.
[3] S. Dekel, Y. Han and P. Petrushev, Anisotropic meshless frames on R^n, J. Fourier Analysis and Applications 15 (2009), 634-662.
[4] G. Kyriazis, K. Park, and P. Petrushev, Anisotropic Franklin bases on polygonal domains, *Math. Nachr.* 279 (2006), 1099-1127.

Directional Constructions in Computational Wave Propagation [M-6A]

Laurent Demanet
Massachusetts Institute of Technology
demanet@gmail.com

We review some multilevel directional constructions that have proven useful for the numerical analysis of wave phenomena. This includes directional cutoffs, the butterfly scheme, and separated wave atom expansions. Low-rank expansions often complement sparsity to make such geometric schemes work in practice. Applications include synthetic aperture radar imaging and seismic imaging.

Approximation of a Function Belonging to Generalized Lipschitz Class by Euler-Cesáro Means of Fourier Series [C-20A]

Binod Prasad Dhakal
Butwal Multiple Campus, Tribhuvan University, Nepal
binod_dhakal2004@yahoo.com

The degree of approximation of functions belonging to $Lipa, Lip(a,p)$, and $Lip(t),p)$ class by Cesaro, Narlund, Euler and matrix summability method has been determined by number of researchers of Modern Analysis. Most of the summability methods are derived from matrix method. In this paper, I have taken product of two summability methods, Euler and Cesaro; and establish a new theorem on the degree of approximation of the function f belonging to $W(Lp,(t))$ classes by Euler - Cesaro method.

Methods and New Phenomena of Orthogonal Matrix Polynomials Satisfying Differential Equations [M-11B]

Manuel Domínguez de la Iglesia
Courant Institute of Mathematical Sciences, New York University, U.S.A.
mdi29@cims.nyu.edu

In the last years new families of orthogonal matrix polynomials on the real line have been found satisfying differential equations with matrix coefficients. We give an overview of the techniques that have led to these examples, focusing on new phenomena which are absent in the scalar theory. We will briefly mention the applications that these new families are having in other fields.

Asymptotic Behavior of Carleman Orthogonal Polynomials [M-6B]

Peter Dragnev* and Erwin Miña-Díaz
Indiana-Purdue University and University of Mississippi
dragnevp@ipfw.edu

Let L be an analytic Jordan curve in the complex plane $\mathbf{C}$. Polynomials that are orthonormal with respect to area measure over the interior domain of L were first considered by Carleman, who established a strong asymptotic formula for the polynomials valid on certain open neighborhood of the closed exterior of L. Here we extend the validity of Carleman's asymptotic formula to a maximal open set, every boundary point of which is an accumulation point of the zeros of the polynomials.

Interlacing of Zeros of Polynomials of Non-adjacent Degree from Different Sequences of Orthogonal Polynomials [M-11B]

Kathy Driver
University of Cape Town, South Africa
Kathy.Driver@uct.ac.za

We study the interlacing property of zeros of Gegenbauer polynomials C_n^λ, $\lambda > -\frac{1}{2}$ of non-adjacent degree from sequences corresponding to different values of the parameter λ. Our results extend the classical theorem, proved by Stieltjes, that the zeros of polynomials of non-adjacent degree within any orthogonal sequence $\{p_n\}_{n=0}^\infty$ satisfy interlacing properties. We also prove results for the interlacing of zeros of Laguerre polynomials of non-adjacent degree corresponding to different values of the parameter α.

Multivariate Splines for Sampling Lattices [C-15A]

Mahsa Mirzargar and Alireza Entezari*
University of Florida
entezari@cise.ufl.edu

We present a non-separable multivariate spline construction for general sampling lattices. These splines are constructed with any degree of continuity and inherit the geometry of a sampling lattice from its Voronoi cell. The lattice-shifts of the spline generate a shift-invariant space with guaranteed approximation order. Using the geometric properties of Voronoi polytopes and zonotopes, we present an evaluation algorithm for the proposed splines.

Hurricane Prediction Using Bivariate Splines [C-18A]

Bree Ettinger*, Ming-Jun Lai
Georgia State University, Atlanta, GA, University of Georgia, Athens, GA
bree@math.uga.edu

To predict the path of a hurricane, we want to use a model where both the explanatory and response variables are random surfaces. In our model, we use bivariate splines over triangulations to represent the random surfaces. To create an input surface, we use barometric pressure data from ocean buoys. Then we use the bivariate spline representations to construct a least squares estimate of the regression function based on spatial functional regression model for each ocean buoy. Finally, we fit a surface through our set of solutions to produce response surface. The location of the hurricane is tracked by identifying eye of hurricane, where the barometric pressure is lowest.

Assessment of the Effectiveness of Multidimensional Splines in Numerical Approximation and Isogeometric Analysis [M-8A]

John A. Evans*, Yuri Bazilevs, Ivo Babuška, and Thomas J.R. Hughes
University of Texas at Austin, Austin, TX, USA
evans@ices.utexas.edu

We assess the effectiveness of multidimensional splines as approximating functions utilizing the theory of Kolmogorov n-widths. We focus on the approximation properties of splines with maximal continuity. Numerical algorithms for computing n-widths and sup-infs in a Sobolev space setting are presented based on the solution of two variational eigenproblems. These algorithms result from the application of Galerkins method and Lanczos iteration. A numerical study is conducted in which we compute the n-width and sup-inf for a large class of smooth multi-dimensional splines. This study reveals the near-optimal approximation

properties of smooth spline functions in multi-dimensional domains. We finish this paper with a comparison of the approximability of splines exhibiting maximal continuity and classical C^0 finite element basis functions. This study illustrates potential advantages of employing k-version NURBS in isogeometric analysis.

Green's Functions: Taking Another Look at Kernel Approximation, Radial Basis Functions and Splines [P-4]

Greg Fasshauer
Illinois Institute of Technology, Chicago, IL, USA
fasshauer@iit.edu

The theories for radial basis functions (RBFs) as well as piecewise polynomial splines have now reached a stage of relative maturity as is demonstrated by the recent publication of a number of monographs in either field. However, there remain a number of issues that deserve to be investigated further. For instance, it is well known that both splines and radial basis functions yield "optimal" interpolants, which in the case of radial basis functions are discussed within the so-called native space setting. It is also known that the theory of reproducing kernels provides a common framework for the interpretation of both RBFs and splines. However, the associated reproducing kernel Hilbert spaces (or native spaces) are often not that well understood — especially in the case of radial basis functions.

By linking (conditionally) positive definite kernels to Green's functions of differential operators we obtain new insights that enable us to better understand the nature of the native space as a generalized Sobolev space. An additional feature of our new perspective is the notion of scale built into the definition of these function spaces. Furthermore, we are able to use eigenfunction expansions of our kernels (Mercer's theorem) to make progress on such important questions as stable computation with flat radial basis functions and dimension independent error bounds.

The topics discussed in this talk are drawn from joint work with Fred Hickernell, Guohui Song, and Qi Ye (all IIT), John Riddle (Wheaton College), and Henryk Woźniakowski (Columbia University).

Quadrature Formulas for Functions Defined on Riemannian Manifolds [M-3A]

Frank Filbir*, Hrushikesh N. Mhaskar
Helmholtz Center Munich, California State University
filbir@helmholtz-muenchen.de

In many practical applications, for example document analysis, semi-supervised learning, and inverse problems one is confronted with functions defined on a (Riemannian) manifold M imbedded in a high dimensional ambient space. These functions have to be approximated by using sample values of the function. Due to several restrictions like experimental setup etc. we can hardly assume that the sampling nodes are located on a regular grid. This means we have to come up with an approximation process which can, on the one hand, work with scattered data and, on the other hand, has sufficiently good approximation rate. We consider approximation processes of the form

$$\sigma_L f(x) = \sum_{j=0}^{\infty} H\big(\frac{\ell_j}{L}\big)\ \langle f, \phi_j\rangle \phi_j(x),$$

where H is a suitable filter function and $\{\phi_j\}$ is an orthonormal function system on the manifold M. In order to get an approximation process which has the aforementioned properties it is necessary to construct quadrature formulas with certain degree of exactness. In this talk we will address this problem and we will show how this is related to the problem of constructing well localized kernels on M.

This talk is based on joint work with Hrushikesh N. Mhaskar, Department of Mathematics, California State University, U.S.A.

Best Sufficient Conditions for Sparse Recovery [M-1A]

Simon Foucart
Université Pierre et Marie Curie, Paris, France
foucart@ann.jussieu.fr

We review some of the sparse recovery algorithms used in Compressive Sensing to reconstruct s-sparse vectors $x \in \mathbf{C}^N$ from the mere knowledge of linear measurements $y = Ax \in \mathbf{C}^m$, $m < N$. For each algorithm, we derive improved conditions on the Restricted Isometry Constants of the measurement matrix A that guarantee the success of the recovery. In particular, we point out the sufficient conditions $\delta_{3s} < 1/2$ for Iterative Hard Thresholding and $\delta_{2s} < 3/(4+\sqrt{6}) \approx 0.4652$ for Basis Pursuit.

An Algorithm to Construct 3D Triangles with Circular Edges [C-20A]

Bertrand Belbis, Sebti Foufou*, and Lionel Garnier
Qatar University, Doha, Qatar
sfoufou@qu.edu.qa

Dupin cyclides are non-spherical algebraic surfaces of degree 4, discovered by the French mathematician Pierre-Charles Dupin at the beginning of the 19th century. A Dupin cyclide has a parametric equation and two implicit equations and circular lines of curvature. It can be defined as the image of a torus, a cone of revolution or a cylinder of revolution by an inversion. A torus has two families of circles: meridians and parallels. A ring torus has one more family of circles called Villarceau circles. As the image, by an inversion, of a circle is a circle or a straight line, there are three families of circles onto a Dupin cyclide too. The goal of this paper is to construct, onto a Dupin cyclide, 3D triangles with circular edges: a meridian arc, a parallel arc and a Villarceau circle arc.

Polynomial Interpolation and New Asymptotic Formulae for Zeta Functions [M-1B]

Michael I. Ganzburg
Hampton University
michael.ganzburg@hamptonu.edu

In this talk we first present explicit formulae for the Lagrange and Hermite interpolation errors of $f(y) = \int_{\mathbf{R}} \frac{d\mu(t)}{t-iy} : \mathbf{R} \to \mathbf{C}$. Next, we use these formulae in the special case of $f(y) = |y|^s$, where s is a complex number, to establish various asymptotic representations for zeta functions $\zeta(s)$ and $\beta(s)$. As a corollary, we obtain new criteria for $\zeta(s) = 0$.

Active Geometric Wavelets [M-13A]

N. Dyn, S. Dekel, I. Gershtansky*
Tel-Aviv University, GE Healthcare, Tel-Aviv University
gershtan@post.tau.ac.il

In this talk we present an algorithm for sparse representations of images that combines the active contour [1] and the adaptive geometric wavelet [2] methods. The algorithm overcomes the limitation of the active contour method that can only segment between a limited number of regions and generalizes the geometry of adaptive wavelet representations beyond dyadic cubes and triangulations. Sample applications will be presented.

[1] T. Chen and L. Vese, Active Contours without edges, IEEE Trans. Image Processing 10 (2001), 266-277.

[2] S. Dekel and D. Leviatan, Adaptive multivariate approximation using binary space partitions and geometric wavelets, SIAM Journal on Numerical Analysis 43 (2005), 707 732.

A Survey of Sparse Approximation [P-7]

Anna Gilbert
University of Michigan, Ann Arbor
annacg@umich.edu

The past 10 years have seen a confluence of research in sparse approximation amongst computer science, mathematics, and electrical engineering. Sparse approximation encompasses a large number of mathematical, algorithmic, and signal processing problems which all attempt to balance the size of a (linear) representation of data and the fidelity of that representation. I will discuss several of the basic algorithmic problems and their solutions and the application of these methods to medical imaging problems.

A Continuous Approach for Distributing Points on the Sphere Using Fast Fourier Transforms [C-18B]

Manuel Gräf*, Daniel Potts
Chemnitz University of Technology, Germany
m.graef@mathematik.tu-chemnitz.de

In this talk we present a spectral method for solving the so-called phase-field-crystal (PFC) model on the sphere. We solve the corresponding 6th order nonlinear partial differential equation by means of nonequispaced fast spherical Fourier transforms [3]. The PFC model was introduced for simulating crystal growth phenomena in flat space [2]. In [4] the authors have shown how this model can be derived after some approximations from a microscopic Smoluchowski equation via dynamical density functional theory. Transfering the PFC model to the sphere one obtains a continuous model of crystal growth which is related to the famous Thomson problem [1]. This problem asks for the position of N electrons on the sphere with minimal Coulomb energy.

[1] R. Backofen, T. Wikowski, and A. Voigt. Particles on curved surfaces - a dynamic approach by a phase-field-crystal method. *Phys. Rev. E*, accepted.
[2] K. R. Elder, M. Katakowski, M. Haataja, and M. Grant. Modeling elasticity in crystal growth. *Phys. Rev. Lett.*, 88:245701, 2002.
[3] J. Keiner, S. Kunis, and D. Potts. Using NFFT3 - a software library for various nonequispaced fast Fourier transforms. *ACM Trans. Math. Software*, 36:Article 19, 1 − 30, 2009.
[4] S. van Teeffelen, R. Backofen, A. Voigt, and H. Löwen. Derivation of the phase-field-crystal model for colloidal solidification. *Phys. Rev. E*, 79:051404, 2009.

Aerospace Applications of Isogeometric Analysis [M-11A]

Thomas A. Grandine
The Boeing Company, Seattle, WA, USA
thomas.a.grandine@boeing.com

For the better part of 20 years, Boeing has been making substantial use of B-splines as finite elements to solve a variety of engineering problems that frequently arise. B-spline finite elements are very appealing for many important reasons which include high-order accuracy, smoothness, and computational ease. They are also appealing because they naturally lead to problem solutions which interact beautifully with much existing computer-aided design (CAD) software. More recently, we have begun to examine isogeometric analysis as an extension to this basic idea with a view toward eliminating one of the more difficult bottlenecks that arise in performing design optimization. This talk will outline some of the progress that we have made in building practical tools to leverage this exciting new technology.

Refinable Functions for Composite Dilation Systems [C-16B]

Philipp Grohs
TU Graz
philippgrohs@gmail.com

I will consider the problem of constructing continuous and compactly supported functions which are simultaneously refinable with respect to several dilation matrices. The motivation for studying this problem comes from the recently introduced Composite Dilation Wavelets in general, and Shearlets in particular. For a large class of composite dilation systems we show the existence of refinable functions with nice properties.

Wavelet Analysis Under the Unifying Roof of Nonhomogeneous Wavelet Systems [P-19]

Bin Han
University of Alberta, Edmonton, Alberta, Canada
bhan@math.ualberta.ca

Linked with discretization of continuous wavelet transforms, most wavelets and framelets studied in the literature are homogeneous wavelet systems generated by square integrable functions. However, in this talk, we show that rarely-studied nonhomogeneous wavelet systems, in particular with wavelet generators being distributions in the frequency domain, play a fundamental role by naturally linking many aspects of wavelet analysis together. In this talk, we shall present results showing that 1. a nonhomogeneous wavelet system naturally leads to multiresolution analysis and refinable function vectors; it is also naturally linked to the fast wavelet transform, filter banks, and approximation theory; 2. classical wavelets are known to have isotropic structure and cannot capture singularities other than point singularity in high dimensions. As an application of nonhomogeneous wavelet systems, we show that directionality in high dimensions can be easily achieved by a natural modification to nonhomogeneous tight wavelet frames. Moreover, such directional tight framelets are associated with filter banks and can be implements by fast wavelet transform; 3. many function spaces can be characterized by wavelets and framelets. In this direction, we shall introduce the concept of nonhomogeneous wavelet systems in function spaces. We shall see that the frame property of a nonhomogeneous wavelet system naturally leads to characterization of function spaces by wavelets or framelets. Using B-splines and box-splines, we shall present examples of wavelets and framelets in Sobolev spaces; 4. we introduce a notion of frequency-based nonhomogeneous (and more general nonstationary) wavelet systems in the distribution space. Using this notion, we obtain a complete characterization of wavelets and framelets in function spaces. This also allows us to have a natural one-one correspondence between filter banks constructed by the Oblique Extension Principle (OEP) and wavelets and framelets in the distribution space without any a priori condition. This notion enables us to fill the gap between filter banks in the discrete setting and wavelets in the continuum function setting. In conclusion, nonhomogeneous wavelet systems allow us to unify many aspects and to have a better picture on the classical theory of wavelets and framelets. Some parts of the talk are based on or motivated by several joint works with various collaborators such as Ingrid Daubechies, Rong-Qing Jia, Qun Mo, Amos Ron, and Zuowei Shen (in alphabetic order). Papers related to this talk can be found at http://www.ualberta.ca/∼bhan/publ.htm

Stable Approximation on Manifolds with Kernels [C-16A]

Thomas Hangelbroek
Texas A&M University, College Station, Texas, USA
hangelbr@math.tamu.edu

A longstanding drawback of kernel approximation has been the absence of effective bases for the underlying approximation spaces. Kernels are global; this gives rise to global approximants, one of their advantages, but it also means that the natural bases do not scale appropriately as underlying sets of centers grow. Very recently it has been shown that, under quasiuniform conditions, a certain large class of kernels on manifolds has nearly local Lagrange functions and, therefore, a bounded Lebesgue constant. These Lagrange functions provide a powerful basis, having condition numbers bounded independent of the number of centers. This has consequences for many other approximation schemes, allowing one to show, for instance, that L_2 minimization, is stable in L_p.

Best m-term Approximation in Tensor Products of Besov and Sobolev Spaces [C-16B]

Winfried Sickel, Markus Hansen*
Friedrich-Schiller-Universität Jena, Germany
markus.hansen2@uni-jena.de

We consider the problem of best m-term approximation with respect to tensor product wavelet bases. This will be done in the context of tensor product of Besov and Sobolev spaces. Special emphasis is given to certain limiting cases which have not been treated before. At the end we shall compare our results to those for entropy numbers and linear widths.

This work continues research done by Temlyakov, Nietzsche, Bazarkhanov and Dinh Dung.

2D RBF Interpolation on Irregular Domain Through Conformal Transplantation [C-16A]

A. Heryudono
UMass Dartmouth, Dartmouth, MA, USA
aheryudono@umassd.edu

Radial Basis Function (RBF) method for interpolating two dimensional functions with localized features defined on irregular domain is presented. RBF points located inside the domain and on its boundary are chosen such that they are the image of conformally mapped points on concentric circles on a unit disk. On the disk, fast RBF solver to compute RBF coefficients developed by Karageorghis et al. is used. Approximation values at desired points in the domain can be computed through the process of conformal transplantation.

New Coins from Old, Smoothly [M-3B]

Olga Holtz*, Fedor Nazarov, Yuval Peres
UC Berkeley, TU Berlin and IAS; UW-Madison; Microsoft Research
holtz@math.ias.edu

Given a (known) function $f : [0,1] \to (0,1)$, we consider the problem of simulating a coin with probability of heads $f(p)$ by tossing a coin with unknown heads probability p, as well as a fair coin, N times each, where N may be random. The work of Keane and O'Brien (1994) implies that such a simulation scheme with the probability $\Pr_p(N < \infty)$ equal to 1 exists iff f is continuous. Nacu and Peres (2005) proved that f is real analytic in an open set $S \subset (0,1)$ iff such a simulation scheme exists with the probability $\Pr_p(N > n)$ decaying exponentially in n for every $p \in S$. We prove that for $\alpha > 0$ non-integer, f is in the space $C^\alpha[0,1]$ if and only if a simulation scheme as above exists with $\Pr_p(N > n) \leq C(\Delta_n(p))^\alpha$, where $\Delta_n(x) := \max\{\sqrt{x(1-x)/n}, 1/n\}$. The key to the proof is a new result in approximation theory: Let $\mathbf{B}_n$ be the cone of univariate polynomials with nonnegative Bernstein coefficients of degree n. We show that a function $f : [0,1] \to (0,1)$ is in $C^\alpha[0,1]$ if and only if f has a series representation $\sum_{n=1}^\infty F_n$ with $F_n \in \mathbf{B}_n$ and $\sum_{k>n} F_k(x) \leq C(\Delta_n(x))^\alpha$ for all $x \in [0,1]$ and $n \geq 1$. We also provide a counterexample to a theorem stated without proof by Lorentz (1963), who claimed that if some $\varphi_n \in \mathbf{B}_n$ satisfy $|f(x) - \varphi_n(x)| \leq C(\Delta_n(x))^\alpha$ for all $x \in [0,1]$ and $n \geq 1$, then $f \in C^\alpha[0,1]$.

Structured Matrices, Continued fractions, and Root Localization of Polynomials [M-11B]

Olga Holtz*, Mikhail Tyaglov
IAS, UC Berkeley, TU Berlin; TU Berlin
holtz@math.ias.edu

We give an account of various connections between several classes of objects: Hankel, Hurwitz, Toeplitz, Vandermonde and other structured matrices, Stietjes and Jacobi-type continued fractions, Cauchy indices, moment problems, total positivity, and root localization of univariate polynomials. Along with a survey of many classical facts, we provide a number of new results.

The Minimum Surface Area Method for Scattered Data Fitting [C-18A]

Ming-Jun Lai, Qianying Hong*
University of Georgia, Athens, GA
qyhong@math.uga.edu

We will present a new approach for scattered data fitting based on bivariate spline functions. For any scattered data located in a polygonal domain, we find an approximation to these data by minimizing surface area of fitting spline functions. The existence, uniqueness and stability of the minimal surface area splines are studied and an iterative algorithm to compute the fitting splines of minimal surface area is derived. The convergence of the iterative solutions will be established. In addition, the fitting spline of minimum surface area are convergent as the size of the triangulation goes to zero in theory. Finally, a couple of numerical examples are shown to demonstrate the effectiveness of this new data fitting approach.

Müntz Type Theorems on the Half Line [C-10B]

Ágota Horváth
Budapest University of Technology and Economics, Budapest, Hungary
ahorvath@math.bme.hu

Müntz type theorems will be given with respect to the system $\{t^{a_k}w(t)\}$, where w is a weight function on $(0, \infty)$. This is a generaliztion of the results of Boas, Fuchs, Leontev, and Badalyan.

Comparing Information Geometric Curves [M-3A]

Sung Jin Hwang*[1], Steven B. Damelin[2], and Alfred O. Hero[1]
[1]University of Michigan, [2]Georgia Southern University
ssjh@umich.edu

The natural metric space in which to compare different probability densities is an information geometry, introduced by Amari in the early 1980's, endowed with the Fisher information metric.The information geometric viewpoint has been popular in many density approximation applications, including blind source separation (Independent Component Analysis - ICA) and channel coding (Blahut-Arimoto), since it provides geometric insight into the approximations. In the two aforementioned applications a successive approximation generates a sequence of densities that trace out a curve through information geometric space. This presentation will present theory for comparing different information geometric curves.

Geometrical Methods for Adaptive Approximation of Image and Video Data [M-13A]

Armin Iske
University of Hamburg, Germany
iske@math.uni-hamburg.de

This talk introduces a novel geometrical concept for adaptive approximation of images and videos. The resulting compression method relies on an efficient nonlinear approximation algorithm for adaptive extraction of geometrical information from the given signal data, which are then represented by a sparse set of significant pixels. The adaptive approximation scheme works with linear splines over anisotropic Delaunay triangulations (tetrahedralizations) of the significant pixels. The good performace of our compression method is finally demonstrated by several numerical comparisons with relevant competitors, such as JPEG2000 (for images) and MPEG4 (for videos). This talk is based on joint work with Laurent Demaret, Nira Dyn, and Wahid Khachabi.

Curvature Analysis of Frequency Modulated Manifolds in Dimensionality Reduction [M-3A]

Mijail Guillemard, Armin Iske*
University of Hamburg
iske@math.uni-hamburg.de

Recent advances in the analysis of high-dimensional signal data have triggered an increasing interest in geometry-based methods for nonlinear dimensionality reduction (NDR). In many applications, high-dimensional data sets typically contain redundant information, and NDR methods are important for an efficient analysis of their properties. During the last few years, concepts from differential geometry were used to create a whole range of new NDR methods. In the construction of such geometry-based strategies, special emphasis is placed on their interaction with classical and modern signal processing tools (convolution transforms, Fourier analysis, wavelet functions). In particular, one important task is the analysis of the incurred geometrical deformation when applying signal transforms to the elements of a data set. We propose the utilization of frequency modulation maps and modulation manifolds for the construction of particular data sets which are relevant in signal processing and NDR. Moreover, we develop a numerical algorithm for analyzing geometrical properties of the modulation manifolds. In our numerical examples, the geometry-based analysis algorithm is finally applied to two model problems, where relevant geometrical and topological effects are illustrated.

Matching of Point Configurations: An Approach Through Grammians [C-10A]

David Jiménez
Texas A&M University, College Station, Texas, USA
djimenez@tamu.edu

On many computer vision applications, it is often necessary to match a given *image* to one in an indexed set. One way this may done by the identification and comparison of *landmarks* on the image, obtaining then, a point configuration, this is, a discrete set of points in R^d. We analyze the possibility of doing such matching of through the study of the Grammian of the configuration.

Kronecker Products and Sparse Approximation [C-5A]

Sadegh Jokar* and Volker Mehrmann
TU Berlin, Germany
jokar@math.tu-berlin.de

The Kronecker product of matrices plays a central role in mathematics and in applications found in engineering and theoretical physics. In this talk, we discuss different kind of sparsity measures such as restricted isometry property (RIP), null space property (NSP), coherence, spark and their connections and how these measures behave in the case where the matrix is the Kronecker product of matrices.

On the Stability of the Hyperbolic Cross Discrete Fourier Transform [M-11A]

L. Kämmerer
Chemnitz University of Technology, Germany
kaemmerer@mathematik.tu-chemnitz.de

A straightforward discretisation of high dimensional problems often leads to an exponential growth in the number of degrees of freedom. So, computational costs of even efficient algorithms like the fast Fourier transform increase similar.

Hyperbolic crosses as frequency grids allow for a good approximation of functions of appropriate smoothness and decrease the number of used Fourier coefficients strongly. The evaluation of a trigonometric polynomial given by its Fourier coefficients on a hyperbolic cross at the corresponding sparse grid in spatial domain, is called hyperbolic cross discrete Fourier transform [1]. We estimate the condition number of the associated Fourier matrix and show the limitations of this discrete Fourier transform [2].

By changing the spatial domain grid to oversampled rank-1-lattices known from numerical integration we can eliminate the stability problem and evaluate multivariate trigonometric polynomials by one dimensional fast Fourier transforms. In addition, we study reasonable small lattices, which allow for unique reconstruction of trigonometric polynomials from their samples.

[1] G. Baszenski and F.-J. Delvos. A discrete Fourier transform scheme for Boolean sums of trigonometric operators. In C. K. Chui, W. Schempp, and K. Zeller, editors, *Multivariate Approximation Theory IV*, ISNM 90, pages 15 – 24. Birkhäuser, Basel, 1989.
[2] L. Kämmerer and S. Kunis. On the stability of the hyperbolic cross discrete Fourier transform. *Preprint 34, DFG SPP 1324*, http://www.dfg-spp1324.de, 2009.

Discrete Rational Approximation Existence [C-15B]

Franklin Kemp
Collin College, Plano TX
lfkemp@collin.edu

The total degree discrete Remes one point exchange algorithm depends on the property that best uniform rational approximations of total degree (i.e. $p+q=n-1$ with n fixed) exist on a finite point set if and only if they exist on every subset of $n+1$ points (a reference) where their errors equioscillate. As the algorithm exchanges references converging to the best overall total degree rational (it may be the total degree polynomial), it may pass through a reference where a total degree rational's error equioscillates on $n+1$ points, but is unbounded over the interval covering the reference; hence, it doesn't exist. This conditioned is recognized by Sturm's test for roots of the denominator polynomial. We present examples how the algorithm not only finds the best overall total degree rational, but also some total degree rationals that don't exist.

Best l_2 Spline-by-Spline Approximation [C-18A]

Scott N. Kersey
Georgia Southern University
skersey@georgiasouthern.edu

It has been shown (by Lutterkort, Peters and Reif) that the problem of best approximation of a polynomial by one of lower degree (a.k.a., degree reduction) in the $L_2(a,b)$ norm is equivalent to degree reduction in the l_2 norm of the degree-raised coefficients in the Bernstein basis. Noting that we can generally get a better approximation from the space of piecewise polynomials, we are interested in the best approximation of a polynomial by a spline function of lower degree with interior knots, and more generally, the best approximation of a spline by another with different degree and knot sequence. In particular, we develop the theory of best spline-by-spline approximation in the l_2 norm of the coefficients embedded in certain common minimal spline space, and observe that this problem is NOT the same as in the $L_2(a,b)$ norm, with some minor exceptions. Finally, noting that we can approximate better with variable knots, we investigate the problem of best free-knot spline-by-spline l_2 approximation (which is therefore different than the $L_2(a,b)$ theory that has been well-studied in the literature).

An Algorithm for Wavelet-Based Elemental Spectrum Analysis [C-16B]

Bruce Kessler
Western Kentucky University, Bowling Green, KY, US
bruce.kessler@wku.edu

At the previous Approximation Theory XII meeting, I discussed some preliminary work with the Applied Physics Institute at Western Kentucky University in using multiwavelets to provide an objective analysis of gamma-ray spectrum generated from fast neutron bombardment of objects, for the purpose of identifying the elemental composition of the object. The method discussed at the time worked moderately well with the limited amount of data provided, but subsequent use with data sets of different compounds and with different detectors brought to light serious flaws with its implementation.

This talk will illustrate those issues and will address how they have been corrected in the current implementation. The algorithm has potential commercial application in analyzing gamma-ray, X-ray, fluorescence, etc. spectra, and is protected by provisional patent. It is currently being used in the grant-funded development of a prototype device for analyzing the active/inactive status of small (less than 2 kg in weight) unexploded ordnance.

Markov-Nikolskii Type Inequalities for Monotone and Monotone Nonnegative Polynomials [M-1B]

Oleksiy Klurman
University of Manitoba, Winnipeg, Canada
lklurman@gmail.com

Let $M_{q,p}(n,k) := \sup_{P_n\in\Pi_n} \|P_n^{(k)}\|_{L_q[-1,1]}/\|P_n\|_{L_p[-1,1]}$, where Π_n denotes the set of algebraic polynomials of degree $\le n$. It is known (A. Markov [1889], V. Markov [1892], E. Hille, G. Szego and J. Tamarkin [1937], S. Nikolskii [1948], I. Daugavet and S. Rafalson [1972], V. Ivanov [1975]) that, for any $0<p,q\le\infty$,

$$M_{q,p}(n,k) \asymp \begin{cases} n^{2k+2/p-2/q}, & \text{if } k>2/q-2/p, \\ n^k(\log n)^{1/q-1/p}, & \text{if } k=2/q-2/p, \\ n^k, & \text{if } k<2/q-2/p. \end{cases}$$

Recently, T. Erdelyi [2009] considered the problem of finding the order of $M_{q,p}^{(l)}(n,k)$ where the supremum is taken over the set of all absolutely monotone polynomials of order l (i.e., polynomials P_n such that

$P_n^{(m)}(x) \geq 0$ for $0 \leq m \leq l$ and $x \in [-1,1]$). In particular, he showed that, for $l \geq k/2$, $k \geq 2$, $q \geq p$,

$$M_{q,p}^{(l)}(n,k) \asymp \left(n^2/l\right)^{k+1/p-1/q}.$$

In this talk, I'll discuss the analog of the above problems for monotone and monotone nonnegative polynomials as well as the sharp Bernstein inequality for monotone polynomials.

Approximation with Constraints [P-17]

Kirill Kopotun
University of Manitoba, Winnipeg, Canada
kopotunk@cc.umanitoba.ca

Given a function f belonging to a subset V of a (e.g. Banach) space X, we approximate it by the elements of a (linear subspace or nonlinear manifold) $Y \subset X$ which also belong to V, and call this process "constrained approximation of f" (with the constraint V). (Note, however, that this does not describe all possible types of constrained approximation, for instance, onesided and intertwining approximation.) For example, if $X = L_p(K)$, Y is the space of all polynomials of total degree $\leq n$, and V is the set of all convex functions on K, where K is a convex body in R^d, then we arrive at "convex polynomial approximation".

We are interested in determining the exact orders of this type of approximation (exact Jackson-type estimates, various n-widths, comparison to the regular (unconstrained) orders, etc.). In this talk, I will discuss the state of the art of some of the problems (there has been quite a bit of progress in recent years) with a particular emphasis on various open problems in this area.

Growth Behaviour and Zero Distribution of Rational Approximants [M-8B]

H. P. Blatt, R. K. Kovacheva*
Institute of Mathematics and Informatics, Bulg Acad of Sci, Sofia BG
rkovach@math.bas.bg, r_kovacheva@hotmail.com

We investigate the growth and the distribution of zeros of rational uniform approximations with numerator degree $\leq n$ and denominator degree $\leq m_n$ for meromorphic functions f on a compact set E in the complex plane where $m_n = o(n/\log n)$ as $n \to \infty$. We obtain a Jentzsch-Szegő type result, i. e., the zero distribution converges weakly to the equilibrium distribution of the maximal Green domain $E_{\rho(f)}$ of meromorphy of f if f has a singularity of multivalued character on the boundary of $E_{\rho(f)}$. The paper extends results for polynomial approximation and rational approximation with fixed degree of the denominator. As applications, Pad approximation and real rational best approximants are considered. The results are obtained jointly with H.-P. Blatt.

An Optimal Family of Exponentially accurate One-Bit Quantization Schemes [C-10A]

Percy Deift, Sinan Güntürk, and Felix Krahmer*
Hausdorff Center for Mathematics, University of Bonn, Bonn, Germany
felix.krahmer@hcm.uni-bonn.de

Sigma-Delta modulation is a popular method for analog-to-digital conversion of bandlimited signals that employs coarse quantization coupled with oversampling. The standard mathematical model for the error analysis of the method measures the performance of a given scheme by the rate at which the associated reconstruction error decays as a function of the oversampling ratio λ. It was recently shown that exponential accuracy of the form $O(2^{-r\lambda})$ can be achieved by appropriate one-bit Sigma-Delta modulation schemes. By general information-entropy arguments r must be less than 1. The current best known value for r is approximately 0.088. The schemes that were designed to achieve this accuracy employ the "greedy" quantization rule coupled with feedback filters that fall into a class we call "minimally supported". In this talk, we discuss the minimization problem that corresponds to optimizing the error decay rate for this class of feedback filters. We solve a relaxed version of this problem exactly and provide explicit asymptotics of the solutions. From these relaxed solutions, we find asymptotically optimal solutions of the original problem, which improve the best known exponential error decay rate to $r \approx 0.102$. Our method draws from the theory of orthogonal polynomials; in particular, it relates the optimal filters to the zero sets of Chebyshev polynomials of the second kind.

Sparse Image Representation by Tetrolet Transform [M-6A]

Jens Krommweh
University of Duisburg-Essen, Germany
jens.krommweh@uni-due.de

We present a new adaptive Haar wavelet algorithm, the so called tetrolet transform, which we have introduced in a previous paper. The construction is similar to the idea of digital wedgelets where Haar functions on wedge partitions are considered. We compute a tetromino partition of the image which takes the local image structures into account. Tetrominoes are shapes made by connecting four equal-sized squares, each joined together with at least one other square along an edge. On these geometric shapes we define Haar-type wavelets, called *tetrolets*, which form a local orthonormal basis. The tetrolet transform has an underlying multiresolution analysis structure and generates data dependent scaling spaces and wavelet spaces.

In the corresponding filter bank algorithm the input image is decomposed by a tetromino partition that is adapted to the local image geometry. After a suitable shrinkage procedure we get an excellent data approximation. We analyze the method regarding multiresolution structure, approximation order, adaptivity costs, and suitable postprocessing steps (e.g. anisotropic total variation minimization). Some numerical results show the efficiency of the tetrolet transform for image approximation.

Approximation and Entropy Numbers in Sequence and Function Spaces [C-5B]

Thomas Kühn
Universität Leipzig, Germany
kuehn@math.uni-leipzig.de

In recent years, the compactness of Sobolev type embeddings in many different settings has attracted a lot of attention and studied by many authors. In particular, the "degree" of compactness of such embeddings, measured in terms of approximation and entropy numbers, has been investigated intensively. The main strategy in all these papers was a reduction of the problem, using wavelet bases of the corresponding function spaces, to the simpler context of sequence spaces.

In the talk I will present some new results on approximation and entropy numbers of (bounded linear) operators in quasi-Banach spaces, more precisely

- a general relation between approximation and entropy numbers,
- an optimal estimate for entropy numbers of diagonal operators in ℓ_p–spaces, and
- some applications to Sobolev type embeddings of weighted spaces, and spaces of generalized smoothness.

These results, partially obtained in collaboration with Fernando Cobos (UCM Madrid), are taken from two recent articles in J. Approx. Theory (2008 and 2009).

On the Butterfly Approximation Scheme for Fourier Transforms [M-11A]

Stefan Kunis
Chemnitz University of Technology, Germany
kunis@mathematik.tu-chemnitz.de

Recently, the so-called butterfly approximation scheme [2] has been proposed for combining a sequence of low rank approximations from appropriate sub-domains. Its applications include a new variant of the nonequispaced fast Fourier transform [1] as well as the sparse Fourier transform for highdimensional data [3]. For this setting, we present an error analysis of the butterfly approximation scheme relying on low rank interpolation with fixed complex exponential functions. We conclude by studying the numerical stability and the arithmetic complexity of the scheme and show some applications within biomedical imaging.

[1] J. Keiner, S. Kunis, and D. Potts, Using NFFT 3 - a software library for various nonequispaced fast Fourier transforms, ACM Trans. Math. Software 36 (2009), 1-30.
[2] M. O'Neil, F. Woolfe, and V. Rokhlin, An algorithm for the rapid evaluation of special function transforms, Appl. Comput. Harmon. Anal., in press.
[3] L. Ying, Sparse Fourier transform via butterfly algorithm, SIAM J. Sci. Comput. 31 (2009), 1678–1694.

Compactly Supported Shearlets:
Construction and Optimally Sparse Approximation [M-6A]

G. Kutyniok* and W.-Q Lim
University of Osnabrueck
kutyniok@uni-osnabrueck.de

Many important problem classes are governed by anisotropic features such as singularities concentrated on lower dimensional embedded manifolds. While the ability to reliably capture and sparsely represent anisotropic structures is obviously the more important the higher the number of spatial variables is, the principal difficulties arise already in two spatial dimensions and even there are yet far from being understood.

Three years ago, shearlets were introduced as a means to sparsely encode anisotropic singularities of 2D data in an optimal way, while – in contrast to previously introduced directional representation systems – providing a unified treatment of the continuous and digital world. One main idea is to parameterize directions by slope through shear matrices rather than angle, which greatly supports the treating of the digital setting. However, previous constructions excluded the possibility of compactly supported shearlets, which are essential for superior spatial localization.

In this talk, we will discuss a novel construction method for shearlet frames generated by compactly supported shearlets. Then, a detailed analysis of the achievable frame bounds will be provided. Finally, we will prove that a large class of shearlet frames generated by compactly supported shearlets indeed even provide optimally sparse approximations of cartoon-like images such as the very special class of shearlet frames generated by band-limited shearlets previously studied.

On the Construction of Frames for Spaces of Distributions [C-5B]

G. Kyriazis* and P. Petrushev
University of Cyprus
kyriazis@ucy.ac.cy

We introduce a new method for constructing frames for general distribution spaces and employ it to the construction of frames for Triebel-Lizorkin and Besov spaces on the sphere. Conceptually, our scheme allows the freedom to prescribe the nature, form or some properties of the constructed frame elements. For instance, our frame elements on the sphere consist of smooth functions supported on small shrinking caps.

A Multi-level and Multi-scale Expansion based on Bivariate Spline Functions [M-13B]

Ming-Jun Lai
Department of Mathematics, University of Georgia, Athens, Georgia, U.S.A
mjlai@math.uga.edu

A modified penalized least squares splines is proposed. For any given smooth function over a bounded polygonal domain, we refine a triangulation of the domain uniformly, choose penalized parameters which is decreased by half for each refinement, and compute iteratively modified penalized least squares fitting splines for the given function. We show that the function is the summation of all modified penalized least squares fitting splines and the L^1 norm of f is the summation of the L^2 norms of these fitting splines with the modified penalized terms. This extends the Parseval equality based on orthonormal systems and/or tight frame systems to the setting based on bivariate splines.

Highdimensional Approximation with Sparse Occupancy Trees [C-5A]

P. Binev, W. Dahmen, Ph. Lamby*
University of South Carolina, Columbia, SC, USA
lamby@math.sc.edu

This work is concerned with scattered data approximation in high dimensions. We address the problem in the context of statistical learning: Given data samples (x^i, y^i), $i = 1, \ldots, N$, independently drawn from a distribution ρ on $X \times Y \subset \mathbf{R}^d \times \mathbf{R}$ we want to return for any query point $x \in X$ an estimate for the regression function $f_\rho(x) = E(y|x)$. Here d should be thought of as large, possibly in the hundreds. Our motivation is to explore the potential of multiresolution ideas for high spatial dimension. For this task we develop new algorithms based on an efficient encoding of the training data using what we call *sparse occupancy trees* which are subtrees of the full master tree consisting only of the nodes containing at least one sample of the data site. On the partitions corresponding to the occupancy trees one can define piecewise constant or piecewise linear approximations based on *simplex subdivisions*. We prove the consistency of our schemes, investigate their relation to the k-nearest neighbor approximation and demonstrate their performance with numerical examples.

Interpolatory Biorthogonal Systems [C-15A]

Jian-ao Lian
Prairie View A&M University, Prairie View, Texas, USA
unurbs@yahoo.com

Starting with a compactly supported symmetric interpolatory scaling function, an efficient formulation is introduced to establish its compactly supported and symmetric dual scaling functions with any order of polynomial production. Bithogonal systems are formed when the scaling function is paired with any of its duals.

Sparse Image Representations using the Discrete Shearlet Transform [M-6A]

Wang-Q Lim
University of Osnabrueck, Osnabrueck, Germany
wlim@mathematik.uni-osnabrueck.de

It is now widely acknowledged that analyzing the intrinsic geometrical features of the underlying object is essential in many applications. In order to achieve this, several directional representation schemes have been proposed.

In this talk, I will discuss a novel approach to develop a fast decomposition algorithm for compactly supported shearlets which can provide optimally sparse approximation of piecewise smooth images.

It will be shown that this decomposition algorithm is derived from unified treatment of both continuous and discrete domain so that it explicitly computes the shearlet coefficients and faithfully inherits the nice features from compactly supported shearlets constructed in continuous domain.

Finally, I will show some applications to illustrate the potential of the discrete shearlet transform in many practical applications.

Orthogonal Super Greedy Algorithm and Applications in Compressed Sensing [C-5A]

Entao Liu* and V.N. Temlyakov
University of South Carolina, Columbia, SC, USA
liue@mailbox.sc.edu

General theory of greedy approximation is well developed. Much less is known on how specific features of a dictionary can be used for our advantage. In this talk we discuss incoherent dictionaries. We build a new greedy algorithm which is called Orthogonal Super Greedy Algorithm (OSGA). The OSGA is more efficient than a standard greedy algorithm, Orthogonal Greedy Algorithm (OGA). We show that the rates of convergence of OSGA and OGA with respect to incoherent dictionaries are the same. The greedy approximation is also a fundamental tool for sparse signal recovery. The performance of Orthogonal Multi Matching Pursuit (OMMP), a counterpart of OSGA in the Compressed Sensing setting, is also analyzed under RIP conditions.

Multiple Orthogonal Polynomials on Star-like Sets [M-6B]

A. López
Vanderbilt University, Nashville, TN, USA
abey.lopez@vanderbilt.edu

We analize asymptotic properties of multiple orthogonal polynomials associated with measures supported on disjoint star-like sets in the complex plane. These polynomials are relevant in the study of spectral properties of banded Hessenberg operators with only two non-zero diagonals.

On a Perfect System [M-8B]

G. Lopez Lagomasino*, U. Fidalgo Prieto
Univ. Carlos III de Madrid, Leganes, Spain
lago@math.uc3m.es

In the 1930's, K. Mahler introduced the notion of a perfect system. Recently, we proved that Nikishin systems are perfect. In this talk we will talk on this matter and some of its applications in the context of multiple orthogonality and Hermite-Padé approximation.

Localization and Saturation Results for Durrmeyer Type Operators [C-10B]

Antonio-Jesús López-Moreno* and José-Manuel Latorre-Palacios
University of Jaén, Jaén, Spain
ajlopez@ujaen.es

It is well known that most of the Bernstein and Durrmeyer type operators present good simultaneous approximation properties. That is to say, we have convergence not only for the operators but also for their derivatives. In that case it is possible to analyze the convergence and the saturation classes for such derivatives. We present several techniques to carry this analysis out and we show localization and saturation results for the convergence of the derivatives of classical sequences of Durrmeyer operators. In this talk we summarize the techniques used by the authors and other researchers and we also introduce new results.

Universality Holds in Measure for Compactly Supported Measures [M-11B]

Doron S Lubinsky
Georgia Institute of Technology, Atlanta, GA
lubinsky@math.gatech.edu

Let μ be a measure with compact support, with orthonormal polynomials $\{p_n\}$ and associated reproducing kernels $\{K_n\}$. We show that bulk universality holds in measure in $\{\xi : \mu'(\xi) > 0\}$. More precisely, given ε, $r > 0$, the linear Lebesgue measure of the set of ξ with $\mu'(\xi) > 0$ and for which

$$\sup_{|u|,|v|\leq r} \left| \frac{K_n\left(\xi + \frac{u}{\tilde{K}_n(\xi,\xi)}, \xi + \frac{u}{\tilde{K}_n(\xi,\xi)}\right)}{K_n(\xi,\xi)} - \frac{\sin \pi (u-v)}{\pi (u-v)} \right| \geq \varepsilon$$

approaches 0 as $n \to \infty$. The novelty is that there are no local or global conditions on the measure μ. Previous results have required regularity as a global condition, and a Szegő condition as a local condition.

As a consequence, for a subsequence of integers, universality holds for a.e. ξ. Under additional conditions on the measure, we show universality holds in an L_p sense for all finite $p > 0$.

Exact Solutions of Some Extremal Problems of Approximation Theory [M-6B]

A. L. Lukashov
Fatih University, Istanbul, Turkey
alukashov@fatih.edu.tr

Dedicated to the memory of Franz Peherstorfer

Recently, F. Peherstorfer and his collaborators considered the problem of finding the best approximation of the function sgn(x), $|x| \in [a, 1]$, by Laurent polynomials of degree $(2k-1, 2m-1)$ and the approximation error $L^k_m(a)$. They solved the problem explicitely in terms of a special conformal mapping and found asymptotics of the best approximation polynomials as $m \to \infty$.

In the talk an analogous explicit solution will be given for the problem of the best approximation of the function sgn(x), $|x| \in [a, 1]$, by rational functions of the form

$$\frac{c_n x^n + c_{n-1}x^{n-1} + \ldots + c_0}{x^{2k_0-1}(x^2 - x_1^2)^{k_1} \ldots (x^2 - x_p^2)^{k_p}},$$

where $0 < x_1 < \ldots < x_q < a < 1 < x_{q+1} < \ldots < x_p$, and the order of the numerator is even and greater than the order of the denominator.

Explicit solutions of other problems of the best approximation will be considered too.

Shape Preserving Approximation on the Real Line with Exponential Weights [M-1B]

Oleksandr Maizlish
University of Manitoba, Winnipeg, Canada
ummaizli@cc.umanitoba.ca

The well-known "Bernstein approximation problem" deals with the possibility of polynomial approximation on the real line with weights. Given a continuous function $W : \mathbf{R} \to (0, 1]$, the problem is to determine whether the set of all algebraic polynomials is dense in C_W, the space of all continuous functions $f : \mathbf{R} \to \mathbf{R}$ such that $\lim_{x\to\pm\infty} f(x)W(x) = 0$, equipped with the norm $\|f\|_W := \sup_{x\in\mathbf{R}} |f(x)W(x)|$. In this talk, we will discuss the possibility of weighted approximation with constraints, namely, k-monotone and one-sided approximation with Freud's weights $W_\alpha(x) := e^{-|x|^\alpha}, \alpha \geq 1$.

On the Problem of Parameter Estimation in Exponential Sums [M-11B]

F. Filbir, H. N. Mhaskar*, J. Prestin
Helmholtz Institute, Munich, Germany, Cal. St. Univ., LA, CA U.S.A.,
hmhaska@gmail.com

The recovery of signal parameters from noisy sampled data is a fundamental problem in digital signal processing. In this paper, we consider the following question. Let $I \geq 1$ be an integer, $\omega_0 = 0 < \omega_1 < \cdots < \omega_I \leq \pi$, and for $j = 0, \cdots, I$, $a_j \in \mathbf{C}$, $a_{-j} = \overline{a_j}$, $\omega_{-j} = -\omega_j$, and $a_j \neq 0$ if $j \neq 0$. Given finitely many noisy samples of an exponential sum of the form

$$\tilde{x}(k) = \sum_{j=-I}^{I} a_j \exp(-i\omega_j k) + \epsilon(k), \qquad k = -2N, \cdots, 2N,$$

where $\epsilon(k)$ are random variables with mean zero, each in the range $[-\epsilon, \epsilon]$ for some $\epsilon > 0$, determine approximately the frequencies ω_j. We combine the features of several recent works on this subject to suggest a stable algorithm, and provide error bounds in terms of ϵ and N. In the absence of noise, our algorithm yields exact results. This is a preliminary report on a work in progress.

Asymptotics of Polynomials Orthogonal on the Unit Disk with respect to a Positive Polynomial Weight [M-6B]

Erwin Miña-Díaz
University of Mississippi, Oxford, Mississippi, USA
minadiaz@olemiss.edu

We derive asymptotics for polynomials orthogonal over the complex unit disk $D = \{z : |z| < 1\}$ with respect to a weight of the form $|h(z)|^2$, with $h(z)$ a polynomial without zeros in D. The behavior of the polynomials is established at every point of the complex plane.

Optimal Meshes for Finite Elements of Arbitrary Order [M-13A]

Albert Cohen, J.-M. Mirebeau*
UPMC, Paris
jm.mirebeau

Given a function f defined on a bounded domain $\Omega \subset \mathbf{R}^2$ and a number $N > 0$, we study the properties of the triangulation $\mathcal{T}_N$ that minimizes the distance between f and its interpolation on the associated finite element space, over all triangulations of at most N elements. The error is studied in the L^p norm or $W^{1,p}$ norm for $1 \leq p < \infty$ and we consider Lagrange finite elements of arbitrary polynomial order $m - 1$. We establish sharp asymptotic error estimates as $N \to +\infty$ when the optimal anisotropic triangulation is used. These estimates involve invariant polynomials applied to the m-th order derivatives of f. In addition, our analysis also provides with practical strategies for designing meshes such that the interpolation error satisfies the optimal estimate up to a fixed multiplicative constant. We partially extend our results to functions of three or more variables and to functions which are discontinuous along smooth geometrical sets.

Smooth Convex Extensions of Functions [C-15A]

Bernd Mulansky
TU Clausthal, Germany
bernd.mulansky@math.tu-clausthal.de

Let a subset $\Omega \subset IR^n$ and a function $f : \Omega \to IR$ be given. We ask for the existence of a smooth convex extension of f to the convex hull of Ω or to IR^n. This problem is well understood in case of finite Ω: There exists a smooth convex interpolant, even a polynomial, for strictly convex data. Here we consider the existence of (extended-valued or finite) convex extensions for arbitrary sets Ω. Based on earlier results on infinite interpolation, sufficient conditions for extendability by a smooth convex function are formulated.

Total Variation Image Denoising [C-10A]

Peter Ndajah*, Hisakazu Kikuchi, and Shogo Muramatsu
Niigata University, Japan
ndajah@gmail.com

In image restoration literature, there are two main approaches: one is based on a process over image wavelet coefficients, the other is based on a process over image gradients. We treat the second case in this work. In order to restore an image while preserving its edges, we assume the image belongs to the space of bounded variation (BV) since L^1-norms are edge-preserving norms. Image energy is minimized. This energy is composed of an observation term and the total variation (TV) of the image gradient. The image noise is modeled as a least squares problem. The least squares term is regularized by the total variation of the image gradient and subsequent minimization achieves a denoising of the image.

Mixed Operators in Compressed Sensing [M-1A]

M. Herman and D. Needell*
Univ. of California, Los Angeles, and Stanford University
dneedell@stanford.edu

Applications of compressed sensing motivate the possibility of using different operators to encode and decode a signal of interest. Since it is clear that the operators cannot be too different, we can view the discrepancy between the two matrices as noise. The stability of ℓ_1-minimization and greedy algorithms to recover the signal in the presence of *additive noise* is by now well-known. Recently however, work has been done to analyze these methods with noise in the measurement matrix, which generates a *multiplicative noise* term. This framework of generalized perturbations (i.e., both additive and multiplicative noise) extends the prior work on stable signal recovery from incomplete and inaccurate measurements of Candès, Romberg and Tao using Basis Pursuit (BP), and of Needell and Tropp using Compressive Sampling Matching Pursuit (CoSaMP). We show, under reasonable assumptions, that the stability of the reconstructed signal by both BP and CoSaMP is limited by the noise level in the observation. Our analysis extends easily to arbitrary greedy methods.

On Extended Cubatures of Turan Type for the Ball [C-18B]

Dimitar Dimitrov, Hao Nguyen*, Guergana Petrova
Texas A&M University, College Station, Texas, USA
htnguyen@math.tamu.edu

We construct explicitly extended cubatures of Turan type for the unit ball in R^n. In particular, we construct a formula that approximates the integral of a function f over a ball by a linear combination of surface integrals of f, its Laplacians of higher order and its normal derivative. There is no other formula of this type that has higher polyharmonic degree of precision.

Korovkin-type Convergence Results for Non-positive Operators Related to a Class of Scattered Data Interpolation Operators [C-10B]

Oliver Nowak
ETH Zurich, Switzerland
oliver.nowak@sam.math.ethz.ch

Korovkin-type approximation theory usually deals with convergence analysis for sequences of positive operators. In this talk we present Korovkin-type convergence results for certain sequences of non-positive operators, namely regular operators with vanishing negative parts. Those sequences, called sequences of almost positive linear operators, appear for instance in the context of moving least squares interpolation, an approach that allows the arbitrarily smooth interpolation of multivariate scattered data.

Local Lagrange Interpolation by Splines on Tetrahedral Partitions [M-13B]

Günther Nürnberger
University of Mannheim, Germany
nuern@staff.mail.uni-mannheim.de

Local Lagrange interpolation methods for cubic C^1 splines on arbitrary tetrahedral partitions are described. Our special aim is to develop algorithms with low locality. For doing this, we construct uniform noncube partitions which can be decomposed into few classes of tetrahedra. This implies that the methods are local and stable, and therefore posses optimal approximation order. Similar local Lagrange interpolation schemes are developed for C^2 splines of degree nine. The numerical and graphic results confirm the efficency of the algorithms.

The Best Approximation of Periodic Functions by Splines [M-1B]

Vladislav Babenko and Nataliya Parfinovych*
Dnepropetrovsk National University
nparfinovich@mail.ru

Let L_p $(1 \leq p \leq \infty)$ be the spaces of 2π-periodic functions $f : R \to R$ with the corresponding norms $\|f\|_p$. Given $r \in N$ and rearrangement invariant set $F \subset L_1$ denote by W^rF the class of functions $f \in L_1$ such that $f^{(r-1)}$ is locally absolutely continuous and $f^{(r)} \in F$. By W^rH^ω let us denote the class of r times continuously differentiable 2π-periodic functions such that $f^{(r)}$ has prescribed concave majorant $\omega(t)$ of modulus of continuity. Let $S^k_{2n,m}$ $(k = 1, 2;\ n, m \in N)$ be the spaces of 2π-periodic polynomial splines of order m and defect k, with knots $\frac{kj\pi}{n}$, $j \in Z$.

By $E(M,H)_p$ and $d_n(M, L_p)$ denote the best L_p-approximation of the class $M \subset L_p$ by the set $H \subset L_p$ and Kolmogorov n-width of this class in the space L_p respectively.

It is well known that the spaces $S^1_{2n,m}$ $(m \geq r)$ and $S^2_{2n,m}$ $(m \geq r + 1)$ are the extremal spaces for the widths $d_{2n}(W^rF, L_1)$ and $d_{2n}(W^rH^\omega, L_1)$.

For $h \in (0, \frac{2\pi}{n})$, by $S^1_{2n,m}(h)$ denote the space of 2π-periodic polynomial splines of order m and defect 1, with knots $\frac{2j\pi}{n}$ and $\frac{2j\pi}{n} + h$, $j \in Z$.

We found the exact values of the the quantities $E(W^rF, S^1_{2n,m}(h))_1$ and $E(W^rH^\omega, S^1_{2n,m}(h))_1$. In addition we proved that the spaces $S^1_{2n,m}(h)$ are also extremal spaces for the widths $d_{2n}(W^rF, L_1)$ and $d_{2n}(W^rH^\omega, L_1)$

(for all $m \geq r+1$).

Theorem. Let $r, n, m \in N$, $k = 1, 2$, $h \in (0, \frac{2\pi}{n})$, let F be an arbitrary rearrangement invariant set of 2π-periodic functions, and let $\omega(t)$ be a concave modulus of continuity. Then for all $m \geq r+1$

$$E(W^r F, S^1_{2n,m}(h))_1 = E(W^r F, S^k_{2n,m})_1 = d_{2n}(W^r F, L_1);$$

$$E(W^r H^\omega, S^1_{2n,m}(h))_1 = E(W^r H^\omega, S^k_{2n,m})_1 = d_{2n}(W^r H^\omega, L_1).$$

Isogeometric Analysis based on Tensioned B-splines in Advection-diffusion Problems [M-8A]

Carla Manni, Francesca Pelosi*, Maria Lucia Sampoli
University of Rome "Tor Vergata", Italy
pelosi@mat.uniroma2.it

NURBS-based isogeometric analysis can be seen as a superset of FEM and offers a powerful tool in the area of numerical methods for differential problems [3]. The NURBS-based approach leads to remarkable results, not only in the context of structural analysis, but also in advection dominated flow phenomena, often characterized by sharp layers involving very strong gradients.
As for the classical FEM approach, "stabilization methods" have to be used in conjunction to NURBS-based isogeometric analysis to profitably face advection dominated flow phenomena.
In this talk we present suitable spaces of B-spline like functions which can efficiently replace NURBS for the previously mentioned problems. The distinguishing property of such spaces is their ability to efficiently describe sharp variations without introducing extraneous oscillations. Thus, they are often referred to as tensioned B-splines.
We focus our attention on a suitable class of tensioned B-splines, possessing a polynomial structure, the so called *variable degree polynomial splines*, VDPS for short [1,2]. We present a VDPS-based isogeometric analysis and we analyze its performances for advection dominated flow phenomena. For the sake of clearness, first, we address the one dimensional case where we remarkably obtain that no stabilization is needed. Then, we present possible applications to two dimensional problems.

[1] Costantini P.; Curve and surface construction using variable degree polynomial splines. Comput. Aided Geom. Design, (2000) **24**, 426–446.
[2] Costantini P. and F. Pelosi; Shape preserving approximation of spatial data, *Adv. Comp. Math.,* **20** (2004), 25–51.
[3] Hughes T.J.R., Cottrell J.A., Bazilevs Y.; Isogeometric analysis: CAD, finite elements, NURBS, exact geometry and mesh refinement, in *Comput. Methods Appl. Mech. Engrg.,* **194** (2005), 4135–4195.

Paley-Wiener and Multiscale Approximations on Manifolds [M-3A]

Isaac Pesenson*, Meyer Pesenson
Temple University, Philadelphia, PA, USA
pesenson@temple.edu

An approximation theory by bandlimited functions ($\equiv$ Paley-Wiener functions) on Riemannian manifolds of bounded geometry is developed. Based on this theory multiscale approximations to smooth functions in Sobolev and Besov spaces on manifolds are obtained. The results have immediate applications to the filtering, denoising and approximation and compression of functions on manifolds. There exist applications to problems arising in data dimension reduction, image processing, computer graphics, visualization and learning theory.

How Fast do Radial Basis Function Interpolants of Analytic Functions Converge? [C-16A]

Rodrigo B. Platte
Arizona State University, Tempe, AZ
Rodrigo.Platte@asu.edu

The question in the title is answered using tools of potential theory. Convergence and divergence rates of interpolants of analytic functions on the unit interval are analyzed. The starting point is a complex variable contour integral formula for the remainder in RBF interpolation. We study a generalized Runge phenomenon and explore how the location of centers and affects convergence. Special attention is given to Gaussian and inverse quadratic radial functions, but some of the results can be extended to other smooth basis functions. Among other things, we prove that, under mild conditions, inverse quadratic RBF interpolants of functions that are analytic inside the strip $|\Im(z)| < (1/2\varepsilon)$, where ε is the shape parameter, converge exponentially.

Impossibility of Approximating Analytic Functions from Equispaced Samples at Geometric Convergence Rate [C-15B]

Rodrigo B. Platte
Arizona State University, Tempe, AZ
platte@math.asu.edu

It is shown that no stable procedure for approximating functions from equally spaced samples can converge geometrically for analytic functions. The proof combines a Bernstein inequality of 1912 with an estimate due to Coppersmith and Rivlin in 1992. This work is in collaboration with L.N. Trefethen and A.B. Kuijlaars.

Optimally Sparse Image Representation by the EPWT [M-13A]

Gerlind Plonka
University of Duisburg-Essen, Duisburg, Germany
gerlind.plonka@uni-due.de

The Easy Path Wavelet Transform (EPWT)has recently been proposed as a tool for sparse representations of bivariate functions from discrete data, in particular from image data. The EPWT is a locally adaptive wavelet transform. It works along pathways through the array of function values and it exploits the local correlations of the given data in a simple appropriate manner. In this talk, we show that the EPWT leads, for a suitable choice of the pathways, to optimal N-term approximations for piecewise Hölder continuous functions with singularities along curves.
This talk is based on joint work with Armin Iske and Stefanie Tenorth.

Surface Reconstruction via L_1-minimization [M-3B]

Bojan Popov
Texas A&M University
popov@math.tamu.edu

In this talk we will consider a class of L_1-based minimization methods for solving (i) digital elevation maps for natural and urban terrain; (ii) super-resolution problems. We will describe each of the two situations. A minimization algorithm based on interior point iteration will be described and various numerical examples will be presented.

Quadrature Rules on Spherical Triangles [C-18B]

J. Beckmann, H. N. Mhaskar and J. Prestin*
University of Lübeck, Lübeck, Germany
prestin@math.uni-luebeck.de

In this talk we present the construction of quadrature rules on arbitrary triangulations of the sphere which are exact for polynomials of some fixed degree. In a first step we study quadrature on some preassigned nodes so that we are able to compute integrals over triangles for arbitrary polynomials. In a second step we apply Cholesky decomposition methods to obtain the weights for scattered data. For our numerical tests we used Mathematica where we carried out all calculations in high accuracy or even with exact numbers. So we were able to overcome a lot of instability problems particularly for very small and thin triangles. Finally, we compare our local quadrature rules on triangulations and some small polynomial degree of exactness with global formulas on the whole sphere and high degree of polynomial exactness. Particularly, for clustered data the local methods seem to be better.

Convexity, Moduli of Smoothness and a Jackson-type Inequality [M-3B]

Z. Ditzian and A. Prymak*
University of Manitoba, Winnipeg, Manitoba, Canada
prymak@gmail.com

For a Banach space B of functions which satisfies for some $m > 0$

$$(*) \qquad \max(\|F+G\|_B, \|F-G\|_B) \geq (\|F\|_B^s + m\|G\|_B^s)^{1/s}, \quad \forall\, F, G \in B$$

a significant improvement for lower estimates of the moduli of smoothness $\omega^r(f,t)_B$ is achieved. As a result of these estimates, sharp Jackson inequalities which are an improvement of the classical Jackson type inequality are derived. Our investigation covers Banach spaces of functions on R^d or T^d for which translations are isometries or on S^{d-1} for which rotations are isometries. Results for C_0 semigroups of contractions are derived. As applications of the technique used in this work, many new results are deduced. An L_p space with $1 < p < \infty$ satisfies $(*)$ where $s = \max(p, 2)$, and many Orlicz spaces are shown to satisfy $(*)$ with appropriate s.

Filling Polygonal Holes Using Minimal Energy Macro-Elements [C-18A]

Vera Rayevskaya
University of Northern Iowa
vera.rayevskaya@uni.edu

We present n-sided macro-elements based on Clough-Tocher triangle splits subject to minimal energy conditions. This construction is used to fill holes in polygonal domains with an arbitrary number of sides. A number of numerical examples are presented to show how the minimal energy n-sided macro-elements of polynomial degree three and C^1 smoothness can be used to fill polygonal holes in surfaces over planar domains.

Efficient Quadrature and Collocation Techniques for Isogeometric Analysis [M-8A]

F. Auricchio, L. Beirão da Veiga, T. J. R. Hughes, A. Reali*, G. Sangalli
Univarsity of Pavia, Italy, and University of Texas at Austin, USA
alessandro.reali@unipv.it

In this work, we show an initial study on two important topics in the framework of NURBS-based isogeometric analysis (see [1,2]), i.e., the construction of efficient quadrature rules (see [3]) and the development of collocation methods (see [4]). In fact, an issue to be definitely considered when dealing with isogeometric analysis is the study of efficient quadrature techniques. In this context, a rule of thumb emerges, i.e., the "half-point rule", indicating that optimal rules involve a number of points roughly equal to half the number of degrees-of-freedom, or, equivalently, half the number of basis functions of the space under consideration. The half-point rule is independent of the polynomial order of the basis. Efficient rules require taking into account the precise smoothness of basis functions across element boundaries. Several rules of practical interest are obtained, and a numerical procedure for determining efficient rules is presented. We also initiate the study of collocation methods for NURBS-based isogeometric analysis. The goal is to combine the accuracy of isogeometric analysis with the low computational cost of collocation to develop accurate and efficient procedures for large-scale structural dynamics and wave propagation problems. To this end, we study some simple model problems and show encouraging numerical results in 1D, 2D and 3D.

[1] T.J.R. Hughes, J.A. Cottrell, Y. Bazilevs. Isogeometric analysis: CAD, finite elements, NURBS, exact geometry, and mesh refinement. *Computer Methods in Applied Mechanics and Engineering*, 194, 4135-4195, 2005.

[2] J.A. Cottrell, T.J.R. Hughes, Y. Bazilevs. *Isogeometric Analysis. Towards integration of CAD and FEA*. Wiley, 2009.

[3] T.J.R. Hughes, A. Reali, G. Sangalli. Efficient Quadrature for NURBS-based Isogeometric Analysis. *Computer Methods in Applied Mechanics and Engineering*, 199, 301–313, 2010.

[4] F. Auricchio, L. Beirão da Veiga, T.J.R. Hughes, A. Reali, G. Sangalli. Isogeometric Collocation Methods. *ICES Report*, 09-30, to appear on *Mathematical Models and Methods in Applied Sciences*, 2010.

$h-p-k$ Approximation Estimates for NURBS [M-8A]

L. Beirão da Veiga, A. Buffa, J. Rivas*, G. Sangalli
Universidad del País Vasco, Bilbao, Spain
judith.rivas@ehu.es

New results on the approximation properties of two-dimensional Non-Uniform Rational B-splines (NURBS) spaces will be presented.

More precisely, we will define a new projection operator into certain spaces of NURBS and give arror estimates in Sobolev norms which are explicit in the three discretization parameters: mesh size, h, degree p and regularity k.

We firstly construct a one-dimensional projector onto the space of splines of degree p with continuous derivatives up to the order $k-1$, with the restriction that $2k-1 \leq p$, and obtain the corresponding error estimates.

The extension to two-dimensional spline approximations is done through a tensor product construction and, finally, the projector onto a NUBRS space is defined by multiplication with the weight and composition with the mapping associated to the considered NURBS space.

These results are a first step in the analysis of the approximation properties of NURBS spaces in terms, not only of the mesh size, but also of the other discretization parameters. However, the restiction over the regularity, k, still leaves the most interesting cases of higher regularity (up to $k = p$) open.

Parameterized Wavelets [C-16B]

David W. Roach
Murray State University
david.roach@murraystate.edu

In this talk, the parameterization of the length-ten wavelets will be presented. Specifically, an explicit parameterization is constructed of the coefficients for all trigonometric polynomials $m(\omega)$ which satisfy the necessary conditions for orthogonality, $m(0) = 1$ and $|m(\omega)|^2 + |m(\omega + \pi)|^2 = 1$, without any vanishing moment restrictions. This class includes all of the one-dimensional scaling functions with their associated wavelets. In particular, length-ten wavelets with only the zeroth vanishing moment are presented that perform better than the standard orthogonal Daubechie's wavelets of equal support and comparable and sometimes better than the biorthogonal 9/7 filter used in the FBI standard in an image compression scheme.

Signal Ensemble Classification on Manifolds [M-3A]

Linh Lieu and Naoki Saito*
University of California, Davis
saito@math.ucdavis.edu

I will discuss our recent effort on classifying ensembles of signals into a set of known classes. Examples of signal ensembles include: sonar waveforms reflecting from underwater objects; hyperspectral image pixels corresponding to different foliations; video sequences of a person speaking different words, etc. Our assumption here is that such signal ensembles are located on different regions on a lower-dimensional manifold in a high-dimensional ambient space. In particular, I will discuss two different classification methods with real examples: (1) Embed signal ensembles into a "diffusion" space, and then classify them by the nearest neighbor method using the Earth Mover's Distance (EMD) as the base distance measure in the diffusion space; (2) Compute transition probability distributions from the diffusion kernel, view them as feature vectors of signal ensembles, and then classify them by the nearest neighbor method using a "statistical" distance of choice (e.g., EMD, the Helliger distance, the Jeffrey's divergence, etc.) among those feature vectors.

Inequalities for the Deviation of Minimal Residual Polynomials and Inverse Polynomial Images [C-20B]

Klaus Schiefermayr
Upper Austria University of Applied Sciences, Wels, Austria
k.schiefermayr@fh-wels.at

Let $\mathbf{P}_n$ denote the set of all polynomials of degree at most n. For a compact infinite set S in the complex plane with $0 \notin S$, consider the following constraint approximation problem: Find that polynomial $P_n^* \in \mathbf{P}_n$ with $P_n^*(0) = 1$, for which the maximum norm on S is minimal, i.e.

$$L_n(S) := \max_{z \in S} |P_n^*(z)| = \min_{P_n \in \mathbf{P}_n, P_n(0)=1} \max_{z \in S} |P_n(z)|.$$

It is known that the limit $\kappa(S) := \lim_{n\to\infty} \sqrt[n]{L_n(S)}$ exists. This approximation problem arise for instance in the context of solving large-scale matrix problems $Ax = b$ by Krylov subspace iterations. In this context, the spectrum $\sigma(A)$ of the matrix A is approximated by a continuum S, i.e. $S \supset \sigma(A)$, where P_n^* is usually called the *minimal residual polynomial* on S and $\kappa(S)$ the *estimated asymptotic convergence factor*. For $\kappa(S)$, there is a nice representation in terms of the corresponding Green's function. The lower bound $L_n(S) \geq \kappa(S)^n$ can be found in many textbooks and papers on the subject. For compact *real* sets S (without any isolated points), we obtain the sharper lower bound

$$L_n(S) \geq \frac{2\,\kappa(S)^n}{1 + \kappa(S)^{2n}}.$$

In the special case of two real intervals, the Green's function and thus $\kappa(S)$ can be given in terms of Jacobian elliptic and theta functions. With the help of this representation, we obtain very accurate estimates for $\kappa(S)$ in terms of *elementary functions* of the endpoints of the intervals. Since each polynomial is (suitable normed) the minimal residual polynomial on its inverse image of $[-1, 1]$, we present some general geometric properties of inverse polynomial images.

Nonuniform Sampling and Recovery of Bandlimited Function via Gaussians [C-5A]

B. A. Bailey, Th. Schlumprecht*, and N. Sivakumar
Texas A & M University
schlump@math.tamu.edu

We approximate multi dimensional bandlimited functions via non uniform sampling by shifts of Gaussian Radial-Basis functions.

Lagrange Interpolation by Trivariate C^2-Splines of Low Locality [M-13B]

G. Nürnberger, G. Schneider*
University of Mannheim, Mannheim, Germany
schneidergh@hotmail.com

We describe a Lagrange interpolation method for C^2-splines of degree nine on a special uniform partition consisting of tetrahedra and octahedra. To ensure the locality of the method, we decompose the partition into classes of tetrahedra. The partition allows an efficient decomposition resulting in few classes. Certain tetrahedra are refined using partial Worsey-Farin splits. We prove the stability of the method, which implies optimal approximation order.

Towards Isogeometric Fluid Analysis in the Design Process of Hydroelectric Turbine Blades [M-11A]

Bert Jüttler, Ulrike Schwarzmair*, Walter Zulehner
JKU Linz, Andritz Hydro, Austria
Ulrike.Schwarzmair@andritz.com

In the frame of the European project EXCITING (see www.exciting-project.eu) we work on the design and optimization of ship propellers and hydroelectric turbine blades. The aim is to improve the current process of designing and developing blades of propellers and turbines by using Isogeometric Analysis. In this talk we will report on our activities and show first results.

In the hydraulic design of turbines, CFD simulations are an important part of the development process. In order to avoid time consuming conversion processes between different geometric models, the turbine blades will be modeled by NURBS only throughout the design process, which gives rise to an isogeometric approach in this context.

In a first step we will replace existing blade surface models governed by shape parameters with clear hydraulic meaning by a consistent NURBS representation. Next we will use this representation for making a volumetric NURBS representation of the surrounding medium of the blades. We will use the Euler equations as the mathematical model for our simulation. The discretization in space will be based on the parametric volume. For stabilization we will use an SUPG method. The resulting semi-discretized system will then be solved with the explicit Euler method.

Local Refinement of Aligned T-spline Spaces [M-8A]

Michael A. Scott*, Thomas W. Sederberg, and Thomas J.R. Hughes
Institute for Computational Engineering and Sciences, Austin, Texas, U.S.A.
mscott@ices.utexas.edu

Isogeometric analysis is an engineering analysis framework capable of interfacing with CAD geometry directly and exactly. It holds great promise to alleviate many current design-through-analysis bottlenecks. To achieve its full potential it requires as input CAD geometry which is analysis-suitable. T-splines overcome the canonical limitations inherent in NURBS while remaining analysis-suitable. T-splines are capable of representing domains of arbitrary topological genus, with high levels of continuity, while maintaining full compatibility with NURBS. T-splines are also trim-free and locally refineable. Unfortunately, the original T-spline local refinement algorithm when applied in an adaptive analysis environment, was shown to be inefficient due to excessive propagation. In fact, it is easy to show that for many simple anisotropic T-mesh configurations refinement propagates globally.

In this work we develop a topological characterization for aligned T-spline spaces and develop a local refinement algorithm for these spaces. This algorithm generates nested T-spline spaces which preserve geometry exactly with minimal dimensionality. We demonstrate how this refinement approach is used in an adaptive Bézier-based isogeometric analysis framework by applying it to several benchmark problems in solids and fluids. Specifically, we focus on examples which are known to trigger global propagation in the original T-spline local refinement algorithm.

Landau-Kolmogorov Inequality Revisited [M-3B]

A. Shadrin
Cambridge University, UK
a.shadrin@damtp.cam.ac.uk

The main guideline in studying the Landau-Kolmogorov inequality in the max-norm on a finite interval is Karlin's conjecture: among all the functions f with $\|f\| = 1$ and $\|f^{(n)}\| = \sigma$, the maximal value $\|f^{(k)}\|$ of the norm of intermediate derivative is attained by an appropriate Zolotarev polynomial or spline. So far, this conjecture has been proved for all $\sigma > 0$ for $n \leq 4$, and for particular $\sigma = \sigma_n$ for all n. Using a new approach, we prove Karlin's conjecture in several further subcases. First of all, we close the "polynomial" case ($\sigma \leq \sigma_n$, all n and k). Secondly, in the spline case, we advanced up to the second intermediate derivative ($k = 1, 2$, all n). And, finally, we obtained nearly complete solution (all σ, almost all k) for (not very) small $n \leq 20$.

On Newton Interpolation and Error Formulas
in Multivariate and Ideal Interpolation [C-10B]

Boris Shekhtman
USF, Tampa, USA
boris@math.usf.edu

In the last twenty or so years a number of open problems had been posed regarding extensions of well-known results in classical (univariate) polynomial interpolation to the multivariate setting. I will present two recent counterexamples to such problems: One is to a conjecture of Carl de Boor regarding existence of certain error formulas; the other is to a conjecture of Tomas Sauer regarding the existence of nested ideal interpolation (Newton interpolation). I will also outline some further question regarding these matters.

The Dimension of the Space of Smooth Splines of Degree 8 on
Tetrahedral Partitions [M-13B]

Xiquan Shi*, Ben Kamau, Hongmin Qu, and Fengshan Liu
Delaware State University, Dover, Delaware, USA.
xshi@desu.edu

Let $V = \{\mathbf{v}_1, \cdots, \mathbf{v}_{n+1}\} \subset R^n$ be a set of $n+1$ points. The convex hull $[V]$ of V is called a n-simplex. $\mathbf{v}_i$ $(1 \leq i \leq n+1)$ are called vertices of $[V]$. $[V]$ is called non-degenerate if its n-dimensional volume is non-zero; otherwise it is called degenerate. The convex hull of $m+1 (m \leq n)$ points of V is also a simplex, called a m-face. A 0-face is called a vertex, a 1-face is called an edge, a 2-face is called a triangle, a 3-face is called a tetrahedron, and a (n-1)-face is called a facet. For a n-simplex σ, we denote $\text{Face}_i(\sigma)$ the collection of all the i-faces of σ.

Definition A simplicial complex Δ in R^n is a collection of simplices in R^n such that every simplex of Δ is a face of a n-simplex of Δ, that every face of a simplex of Δ is also in Δ; and that the intersection of any two simplices of Δ is empty or a face of each other. For convenience, we call a simplicial complex in R^3 a tetrahedral partition.

We denote $\Omega = \bigcup\{\sigma;\ \sigma \in \Delta \text{ is a simplex}\}$ the region covered by the simplicial complex Δ. A simplex $\delta \in \Delta$ is called boundary if $\delta \subset \partial\Omega$, the boundary of Ω. Otherwise, it is called inner (or interior). An inner edge e of Δ is called singular if there are four pairwise coplanar triangles of Δ sharing e as a common edge.

For an i-simplex σ of Δ, we denote

$$\text{Star}(\sigma) = \{\tau \in \Delta;\ \tau \text{ is a face of } \delta \in \Delta, \text{ and } \sigma \text{ is an } i\text{-face of } \delta\}$$

the simplicial complex of the collection of all the simplices (together with their faces) with σ as a common face. $\text{Star}(\sigma)$ is called the i-star of σ.

Let $\Omega \subset R^3$ be a connected polyhedral domain that is allowed to contain polyhedral holes and Δ be a simplical complex of Ω. Given $0 \leq r \leq d$, we define

$$S_d^r(\Delta) = \{s \in C^r(\Omega);\ s|_\sigma \in P_d \text{ for any } \sigma \in \text{Face}_n(\Delta)\}$$

the spline space of degree d and smoothness r, where P_d is the n-variable polynomial space of total degree not exceeding d.

In this paper, we obtained the following results.

Theorem

$$\dim S_8^1(\Delta) = \sum_{\mathbf{v} \in V} \dim S_3^1(\text{Star}(\mathbf{v})) + 5|E| + 9|F| + |T| + 3|E_b| + 3|E_\delta|,$$

where V, E, F, T, E_b, and E_δ are the sets of vertexes, edges, triangles, tetrahedrae, boundary edges, and singular edges of Δ, respectively.

Exact Asymptotics of the Best Asymmetric Piecewise-linear Approximation of Functions with Positive Hessian [M-3B]

V. Babenko[1], Yu. Babenko[2], N. Parfinovych[1], D. Skorokhodov*[1]
[1] Dnepropetrovsk National University, [2] Sam Houston State University
dmitriy.skorokhodov@gmail.com

Let $D = [0,1]^2$; L_p, $1 \le p \le \infty$, be the space of functions $f : D \to \mathbf{R}$ with the usual norm $\|\cdot\|_p$; α, β be positive numbers; $f \in L_p$. Asymmetric L_p-norm is defined as follows

$$\|f\|_{L_{p;\alpha,\beta}(D)} := \|\alpha \max\{f; 0\} - \beta \min\{f; 0\}\|_p.$$

For $N \in \mathbf{N}$ by $\triangle_N = \{T_i\}_{i=1}^N$ denote the triangulation of D consisting of N triangles. Denote by $\mathcal{S}(\triangle_N)$ the space of continuous on D functions, which are linear on every triangle $T_i \in \triangle_N$. Set

$$R_N(f, L_{p;\alpha,\beta}) := \inf_{\triangle_N} \inf_{s \in \mathcal{S}(\triangle_N)} \|f - s\|_{L_{p;\alpha,\beta}(D)}.$$

Let T_0 be the equilateral triangle and let $H(f; x, y)$ be the Hessian of function $f \in C^2(D)$.

We show that for every $f \in C^2(D)$ with positive Hessian

$$\lim_{N\to\infty} N \cdot R_N(f, L_{p;\alpha,\beta}) = C_{p;\alpha,\beta} \left(\int_D H^{\frac{p}{2(p+1)}}(f; x, y)\, dxdy \right)^{\frac{p+1}{p}},$$

$$C_{p;\alpha,\beta} := \inf_{a,b,c \in \mathbf{R}} \|u^2 + v^2 - au - bv - c\|_{L_{p;\alpha,\beta}(T_0)}.$$

Note that, the exact asymptotic behavior of the best interpolation, best approximation and best one-sided approximation of function $f \in C^2(D)$ (with positive Hessian) by piecewise-linear splines $s \in \mathcal{S}(\triangle_N)$ can be derived as particular cases of our result.

Fourier and Rademacher Projections in L_p Spaces [C-15B]

Leslaw Skrzypek
University of South Florida, Tampa, FL, USA
skrzypek@usf.edu

Fourier and Rademacher projections are minimal in L_p spaces. We will present what is known about their unique minimality. As a related topic we will discuss their norming points, norming functionals and their norms.

Intrinsic Supersmoothness of Multivariate Splines [M-13B]

Tatyana Sorokina
Towson University, Towson, MD, USA
tsorokina@towson.edu

We show that many spaces of multivariate splines possess additional smoothness (supersmoothness) at certain faces where polynomial pieces join together. This phenomenon affects the dimension and interpolating properties of splines spaces. The supersmoothness is caused by the geometry of the underlying partition.

Convexity of Spline Functions on Triangulations [C-18A]

Hendrik Speleers*[1], Larry L. Schumaker[2]
[1] Katholieke Universiteit Leuven, [2] Vanderbilt University
hendrik.speleers@cs.kuleuven.be

Convexity is often required in the design of surfaces. Typically, a nonlinear optimization problem arises, where the objective function controls the fairness of the surface and the constraints include convexity conditions. We consider convex polynomial spline functions defined on triangulations. In general, convexity conditions on polynomial patches are nonlinear. In order to simplify the optimization problem, it is advantageous to have linear conditions. We present a simple construction to generate sets of sufficient linear convexity conditions for polynomials defined on a triangle. This general approach subsumes the known sets of linear conditions in the literature. It also allows us to generate sets of convexity conditions that are symmetric.

Asymptotic Properties of Müntz Orthogonal Polynomials [M-11B]

Úlfar F. Stefánsson
Georgia Institute of Technology, Atlanta, GA
ulfar@gatech.edu

Müntz polynomials arise from consideration of Müntz's Theorem, which is a beautiful generalization of Weierstrass's Theorem. We prove a surprisingly simple representation for the Müntz orthogonal polynomials on the interval of orthogonality, and in particular obtain new formulas for some of the classical orthogonal polynomials (e.g. Legendre, Jacobi, Laguerre). This allows us to determine strong asymptotics and endpoint limit asymptotics on the interval, and this yields results on the behavior of the zeros. This is the first time that such asymptotics have been obtained for general Müntz exponents. We also look at the asymptotic behavior outside the interval, and the asymptotic properties of the associated Christoffel functions. If time allows we discuss possible applications in random matrix theory related to so-called biorthogonal ensembles.

Strong Asymptotics for Szegö and Bergman Polynomials over Non-smooth Domains [M-6B]

N. Stylianopoulos
University of Cyprus, Cyprus
nikos@ucy.ac.cy

Let G be a bounded simply-connected domain in the complex plane $\mathbf{C}$, whose boundary $\Gamma := \partial G$ is a rectifiable Jordan curve. We consider the two classical sequences of orthonormal polynomials defined by Γ and G. These are the Szegö polynomials $\{P_n\}_{n=0}^{\infty}$, defined by the arclength measure on Γ and the Bergman polynomials $\{p_n\}_{n=0}^{\infty}$, defined by the area measure on G. That is,

$$\frac{1}{2\pi}\int_{\Gamma} P_m(z)\overline{P_n(z)}\,|dz| = \delta_{m,n}, \quad \text{with} \quad P_n(z) = \mu_n z^n + \cdots, \quad \mu_n > 0,$$

and

$$\int_{G} p_m(z)\overline{p_n(z)}\,dA(z) = \delta_{m,n}, \quad \text{with} \quad p_n(z) = \lambda_n z^n + \cdots, \quad \lambda_n > 0.$$

Also, let $\Omega := \overline{\mathbf{C}}\setminus(\overline{G})$, let Φ denote the conformal map $\Omega \to \{w : |w| > 1\}$, with $\Phi(z) = \gamma z + \gamma_0 + \frac{\gamma_1}{z} + \frac{\gamma_2}{z^2} + \cdots$, $\gamma > 0$, and note that the constant $1/\gamma$ gives the (logarithmic) capacity of Γ.

The purpose of the talk is to report on recent results concerning the strong asymptotics of P_n and p_n in Ω, in cases when Γ is piecewise analytic. More precisely, we will show for the leading coefficients and for any $z \in \Omega$ that

$$\frac{\gamma^{2n+1}}{\mu_n^2} = 1 + O\left(\frac{1}{n}\right), \quad P_n(z) = \Phi^n(z)\sqrt{\Phi'(z)}\left\{1 + O\left(\frac{1}{\sqrt{n}}\right)\right\}$$

and

$$\frac{n+1}{\pi}\frac{\gamma^{2(n+1)}}{\lambda_n^2} = 1 + O\left(\frac{1}{n}\right), \quad p_n(z) = \sqrt{\frac{n+1}{\pi}}\,\Phi^n(z)\Phi'(z)\left\{1 + O\left(\frac{1}{\sqrt{n}}\right)\right\}.$$

These asymptotics complement results of G. Szegö and T. Carleman, from the 1920's, and of P.K. Suetin, from the 1960's, who obtained them by assuming various degrees of smoothness for Γ.

A Hybrid Algorithm for Image Approximation Based on the EPWT [M-13A]

G. Plonka, D. Roşca, S. Tenorth*
University of Duisburg-Essen, Germany and Technical University of Cluj-Napo
stefanie.tenorth@uni-due.de

The EPWT is a locally adaptive wavelet transform that makes use of strong correlations of adjacent pixels. First, a path through all pixels (i.e. a permutation of all pixels) is calculated, so that pixels with similar values are adjacent on the path. Then a one-dimensional wavelet transform (e.g. the biorthogonal Cohen-Daubechies-Feauveau 9/7-transform) is applied to the newly ordered pixels. However, the EPWT suffers from its adaptivity costs that arise from the storage of path vectors.

The proposed hybrid algorithm exploits the advantages of the usual tensor product wavelet transform for the representation of smooth images and uses the Easy Path Wavelet Transform (EPWT) for an efficient representation of edges and texture. It works as follows. The given image is smoothed by a diffusion procedure. A biorthogonal tensor product wavelet transform is applied to the smoothed image. Further, the EPWT is used to construct a sparse representation of the (shrunken) difference image.

Numerical results show the efficiency of this procedure.

This talk is based on joint work with Gerlind Plonka (University of Duisburg-Essen, Germany) and Daniela Roşca (Technical University of Cluj-Napoca, Romania).

Anisotropic Image Regularization Using Double Orientations [M-13A]

G. Steidl and T. Teuber*
University of Mannheim, Germany
tteuber@kiwi.math.uni-mannheim.de

A possibility to improve the quality of image restoration methods is to involve directional information in the restoration process. The aim of this talk is to present a new technique based on anisotropic regularization which is particularly suited to recover sharp corners and X junctions in the presence of heavy noise. To estimate the (smoothed) orientations of the image structures, we apply a model of Aach et al. which finds double orientations at vertices and X junctions. Based on shape preservation considerations this directional information is then incorporated in our final energy functional. We show the relations of our new method to anisotropic diffusion and demonstrate its very good performance by numerical examples.

The Polynomial Inverse Image Method [P-12]

Vilmos Totik
University of South Florida, Tampa, FL, and University of Szeged, Hungary
totik@mail.usf.edu

We discuss a procedure with which one can transform results (not proofs) from an interval or a circle to general sets. The method is based on taking inverse images of intervals or circles under a polynomial map. The essence of the method is that on these model sets, say, on [-1,1] or on the unit circle, strong tools of classical harmonic analysis and approximation theory are available, which do not work on more general sets; however (in certain cases) the method to be discussed is capable to trasform the model result to general ones. It is besed on our ability to approximate general sets by polynomial inverse images. We shall discuss several applications which include sharp polynomial inequalities, asymptotics of Christoffel functions, Bernstein's approximation theorem for —x— and Lubinsky's universality theorem.

Building Quaternionic Hermitian Curves [C-18B]

Mira Bozzini, Leonardo Traversoni*, Rafael Resendiz
Universita degi Studi Milano, Universidad Autonoma Metropolitana Mexico
ltd@xanum.uam.mx

There is extense literature about the use of quaternions in building geometric curves over a sphere for example or for describing a movement of a solid. Our goal is more specific is about building a quaternionic curve that is not a geometric curve but a movement interpolated between movements that constitute the data. One physical example is the behavior and trajectory of an hurricane as well as its strength. All this may be related to a quaternionic variable, the vorticity of which we have several observations in the path of the hurricane and we want to describe all this variables along the trajectory. Our final goal for further work is to build NURBS of quaternions which have more interesting properties, however having a hermitian interpolant is an advance that is better suited for our purpose than the usual quaternionic B-spline.

Sparse Solutions to Linear Inverse Problems

Joel A. Tropp
California Institute of Technology
jtropp@acm.caltech.edu

A fundamental problem in applied mathematics, statistics, and electrical engineering is to solve underdetermined systems of linear equations. Basic linear algebra seems to forbid this possibility. Nevertheless, a recent strand of research has established that certain underdetermined systems can be solved robustly with efficient algorithms, provided that the solution is sparse (i.e., has many zero components). This talk provides an overview of these sparse representation problems, and it describes some of the basic algorithmic approaches. Then it outlines situations where the algorithms are guaranteed to succeed in recovering the sparsest solution to a linear system.

The Gelfand Widths of ℓ_p-balls for $0 < p \leq 1$ [M-1A]

S. Foucart, A. Pajor, H. Rauhut, T. Ullrich*
Hausdorff-Center for Mathematics, Bonn, Germany
tino.ullrich@hcm.uni-bonn.de

In this talk we study the asymptotical behavior of the Gelfand widths of ℓ_p-balls in the N-dimensional ℓ_q^N-space for $0 < p \leq 1$ and $p < q \leq 2$. We provide sharp lower and upper bounds, which are of high relevance to the novel theory of compressive sensing. Our proofs use only techniques from this field.

On Approximation of Piece-Wise Analytic Functions on Finite Interval [M-11A]

V. Vatchev
University of Texas at Brownsville, TX, U.S.
vesselin.vatchev@utb.edu

It is well known that the partial Fourier series of an analytic and periodic function converges exponentially to the function in uniform norm. Recently Gottlieb, Shu, Solomonof, and Vandeven obtained exponential convergence using partial sums of Gegenbauer polynomials for non-periodic functions in uniform norm. In the talk we present a method for approximating real analytic functions(periodic and no-periodic) on a finite interval by linear combinations of the form $\sum_{j=1}^{N} e^{\lambda_j t}$, for a real variable t and complex $\lambda_j = r_j + i\theta_j$. We discus applications of the proposed method in Signal Processing.

Johnson-Lindenstrauss Lemma for Circulant Matrices [C-20B]

A. Hinrichs and J. Vybíral*
RICAM, Linz, Austria
jan.vybiral@oeaw.ac.at

We present two proofs a variant of a Johnson-Lindenstrauss lemma for matrices with circulant structure. This approach allows to minimise the randomness used, is easy to implement and provides good running times. The price to be paid is the higher dimension of the target space $k = O(\varepsilon^{-2} \log^3 n)$ or $k = O(\varepsilon^{-2} \log^2 n)$, respectively, instead of the classical bound $k = O(\varepsilon^{-2} \log n)$.

The possible improvements of this bound on k are left as an open problem for further research.

Uncertainty Principle and Balian-Low Type Theorems in Shift-Invariant Spaces [C-16B]

Akram Aldroubi, Qiyu Sun, and Haichao Wang*
Vanderbilt University
haichao.wang@vanderbilt.edu

In this talk, we consider the time-frequency localization of the orthonormal generator of a shift-invariant space. We prove that if a principal shift-invariant space is translation-invariant then its orthonormal generator is not integrable, and that if a principal shift-invariant space is also $\frac{1}{n}Z$-invariant for some $n > 1$, then the orthonormal generator and its Fourier transform can not both decay as fast as $\frac{c}{x^r}$ for positive constants $r > 1$ and c.

Spline Spaces on Triangulations with Hanging Vertices [C-18A]

Larry L. Schumaker and Lujun Wang*
Vanderbilt University, Nashville, TN
lujun.wang@vanderbilt.edu

Dimension formulae for C^0 Polynomial spline spaces defined on triangulations with hanging vertices are obtained. Explicit basis functions are constructed, and conditions under which they are local and stable are presented.

Asymptotics of Orthogonal Polynomials and Order Reduction Method of Difference Equations [M-8B]

Xiang-Sheng Wang
City University of Hong Kong
xswang4@mail.ustc.edu.cn

A new method is introduced to derive asymptotic formulas for the orthogonal polynomials which satisfy a second order linear difference equation. The main idea is to reduce the second order linear difference equation into a first order nonlinear difference equation. So far as we know, this order reduction method can be applied to reproduce the well known asymptotic formulas of the Hermite polynomials, the Laguerre polynomials and the Legendre polynomials in the outer regions where these polynomials have no zeros. In this talk, we will discuss the possibility of applying this new method to a more general class of orthogonal polynomials.

L^p Bernstein Inequalities and an Inverse Theorem for RBF Approximation on Euclidean d-space [C-16A]

J. P. Ward
Texas A&M University, College Station, TX, USA
johnward@math.tamu.edu

Bernstein inequalities and inverse theorems are a recent development in the theory of radial basis function(RBF) approximation. This talk will briefly cover what is known as well as some new results. In particular, we will discuss L^p Bernstein inequalities for RBF networks on Euclidean d-space. These inequalities involve bounding a Bessel-potential norm of an RBF network by its corresponding L^p norm in terms of the separation radius associated with the network. The Bernstein inequalities will then be used to prove an inverse theorem for RBF approximation.

Sparse Legendre Expansions via l_1 Minimization [M-1A]

Rachel Ward* and Holger Rauhut
*Courant Institute, New York University, New York City, New York, USA
rward314@gmail.com

We extend compressive sensing results concerning the recovery of sparse trigonometric polynomials from few point samples to the recovery of polynomials having a sparse expansion in Legendre basis. In particular, we show that uniform recovery of polynomials whose coefficients are s-sparse in Legendre basis is guaranteed with high probability, using a number of samples that scales linearly with the sparsity level.

Geometric Subdivision Schemes and Interpolatory Multiscale Transforms Between Manifolds [C-20A]

Andreas Weinmann
TU Graz, Austria
andreas.weinmann@tugraz.at

This talk is concerned with manifold-valued subdivision schemes for non-regular combinatorics. We present our results on convergence and smoothness of such geometric schemes. Then we define an interpolatory multiscale transformation for functions on a compact 2-manifold with values in a manifold. We characterize the Hölder-Zygmund smoothness of a function between those manifolds in terms of the coefficient decay w.r.t. this multiscale transform.

The Point Mass Problem – Recent Developments [M-8B]

M. L. Wong
University of Oklahoma
math.wmw@gmail.com

We study the point mass problem, i.e., what happens when one adds a pure point to a probability measure.

Starting from the point mass formula, I will discuss the point mass problem and its connection with the asymptotics orthogonal polynomials.

Recent developments will be presented.

Weak Asymptotics of H^2-best Rational Approximants to Algebraic Functions [M-8B]

L. Baratchart, H. Stahl, and M. Yattselev*
Vanderbilt University
maxim.yattselev@vanderbilt.edu

Let f be an algebraic function holomorphic at infinity with all its singularities contained in the unit disk, $\mathbf{D}$. Let further $\{r_n\}$ be a sequence of L^2-best rational approximants to f on the unit circle. We show that $\{r_n\}$ converges in capacity to f in $\overline{\mathbf{C}} \setminus \mathrm{K}$, the unique domain characterized by the property of minimal condenser capacity of the compact K relative to $\mathbf{D}$, and that the counting measures of the poles of r_n weakly converge to the Green equilibrium distribution on K relative to $\mathbf{D}$.

Green Function Approach to (Conditionally) Positive Definite Function and Reproducing Kernel of Generalized Sobolev Space [C-16A]

Qi Ye
Illinois Institute of Technology, Chicago IL
qye3@iit.edu

In this talk we introduce a generalization of the classical $L_2(\mathrm{R}^d)$-based Sobolev spaces with the help of a vector operator $\mathbf{P}$ which consists of finitely or countably many operators P_n that may be of a differential or even more general type. We find that certain proper full-space Green functions G with respect to $L = \mathbf{P}^{*T}\mathbf{P}$ are (conditionally) positive definite functions. Here we ensure that the vector (distributional) adjoint operator $\mathbf{P}^*$ of $\mathbf{P}$ is well-defined in the distributional sense. We then provide sufficient conditions under which our generalized Sobolev space will become a reproducing-kernel Hilbert space whose reproducing kernel can be computed via the associated Green function G. As an application of this theoretical framework we use G to construct multivariate minimum-norm interpolants $s_{f,X}$ to data sampled from a generalized Sobolev function f on X. Among other examples we show how the Gaussian kernel $K(x,y) := e^{-\sigma^2\|x-y\|_2^2}$ is the reproducing kernel of a generalized Sobolev space.

New Results in Geometric Modeling [C-10A]

Ron Goldman, Plamen Simeonov, and Vasilis Zafiris*
Rice University and University of Houston-Downtown, Houston, Texas
zafirisv@uhd.edu

We present new algorithms for constructing curves and surfaces.

Matrix Extension with Symmetry and Its Applications [C-16B]

Xiaosheng Zhuang
University of Alberta
xzhuang@math.ualberta.ca

The matrix extension problem plays a fundamental role in many areas, especially in wavelet analysis and filter banks design. In this talk, we shall introduce the matrix extension problem: Given several columns of Laurent polynomials satisfy certain conditions, how to extend these given columns to a whole square paraunitary matrix of Laurent polynomials? Without considering any symmetry issue, there are simple algorithms for the matrix extension problem [1]. Yet symmetry is important and sometimes crucial in applications such as image/signal processing. We study the matrix extension problem with symmetry in the most general case [2]. Here are our results we shall talk about:

(I) When the given columns have certain symmetry pattern, our results show that the extension paraunitary matrix also has certain symmetry pattern.

(II) The support of the extension matrix is controlled by the given columns in an optimal way, i.e., it never exceeds the support length of the given columns.

(III) The extension matrix can be written as a product of some simple paraunitary matrices (A cascade structure in engineering).

(IV) A step-by-step algorithm for our matrix extension with symmetry is proposed, which produces a cascade structure for the extension matrix.

To illustrate our results on matrix extension with symmetry, we shall talk about the applications of matrix extension on the construction of tight wavelet frame and on the construction of orthogonal multiwavelets from interpolating orthogonal refinable function vectors [3, 4]. Finally, we shall talk about some related topics to the matrix extension problem such as the biorthogonal matrix extension problem, which is a generalization of the matrix extension problem and is related to the construction of biorthogonal multiwavelets.

[1] W. Lawton, S. L. Lee, and Z. Shen, An algorithm for matrix extension and wavelet construction, Math. Comp., 65 (1996), 723-737

[2] B. Han and X. S. Zhuang, Matrix extension with symmetry and its application to filter banks, preprint

[3] B. Han and X. S. Zhuang, Analysis and construction of multivariate interpoalting refinable function vectors, Acta. Appl. Math. 107 (2009), 143-171

[4] B. Han, S. G. Kwon and X. S. Zhuang, Generalized interpolating refinable function vectors, J. Comp. Appl. Math., 227 (2009), 254-270.

Notes

Notes

Notes