Condensation of Parameters
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It is desirable that spline based multivariate macro elements require only those data necessary to
obtain the prerequisite smoothness. Often there are more coefficients than needed for that purpose.
These can be disposed of in several ways. The process is referred to as condensation of parameters. It
may have the undesirable side effect of reducing the approximation order of the scheme. In this talk I’ll
describe several ways of condensing parameters, some of which maintain the full approximation order
of the macro scheme. The talk is based on joint work with Larry Schumaker and Tanya Sorokina.

On pointwise convergence of continuous wavelet transforms
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For one dimensional continuous wavelet transforms we prove convergence everywhere on the entire
Lebesgue set of $L_p$-functions, $1 \leq p < \infty$. Then we prove the same result for spherically symmetric
wavelet transforms in multidimensional case. We also introduce the Riesz means for continuous spheri-
cally symmetric wavelet transforms and investigate pointwise convergence problems for different classes
of functions.

Jackson-Stechkin type inequalities for best approximations
in Hilbert spaces
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Sharp Jackson-Stechkin type inequalities for best $L_2$-approximations of functions from $L_2(T)$ by
trigonometric polynomials and of functions from $L_2(R)$ by entire functions of exponential type are
well-known and have been extensively studied. There also have been studies of inequalities of such type
for generalized modulus of continuity, as well as for best approximations of elements of Hilbert spaces
by entire vectors of the given self-adjoint operator.

We shall present our recent new Jackson-Stechkin type inequalities with generalized modulus of con-
tinuity for best approximations of elements of Hilbert space by subspaces, generated by given partition
of unity.

Part of the results to be presented have been obtained jointly with S. Savela.

Adaptive approximation by harmonic and polyharmonic splines
on box partitions
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Interpolation by various types of splines is one of the standard procedure in many applications. In
this talk we shall discuss harmonic and polyharmonic spline interpolation as an alternative to polynomial
spline interpolation on box partitions. We will consider some advantages and drawbacks of this approach
and present some error estimates for adaptive approximation by harmonic and polyharmonic splines.

Multivariate polynomial interpolation and sampling in Paley-
Wiener spaces
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In this paper, an equivalence between existence of particular exponential Riesz bases for spaces
of multivariate bandlimited functions and existence of certain polynomial interpolants for these ban-
dlimited functions is given. Namely, polynomials are constructed which interpolate certain classes of
unequally spaced data nodes and corresponding sampled data. Existence of these polynomials allows
one to construct a simple sequence of approximants for an arbitrary multivariate bandlimited function
which demonstrates strong global convergence. A simpler computational version of this recovery for-
mula is also given, at the cost of replacing global convergence with local convergence on increasingly
large domains. Concrete examples of pertinent Riesz bases and unequally spaced data nodes are given.

A smooth shape preserving cubic spline approximation
of a bivarite piece-wise linear interpolant
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We introduce an algorithm for computing a \(C^1\) cubic spline over a planar triangulation preserving
the shape of a piece-wise linear interpolant. The resulting spline does not interpolate the original data.
It depends on a set of parameters controlling "flatness" regions of the spline surface, and proximity of
the spline surface to the original data. In the limit, as the the magnitude of the parameter increases to
infinity, the cubic spline approaches the linear interpolant.

Non-uniform local interpolatory subdivision surfaces
for regular quadrilateral meshes
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Subdivision is a powerful mechanism for generating curves and surfaces from discrete sets of control
points. So far, the main advantage of subdivision methods with respect to other free-form representa-
tions, such as splines, has been acknowledged in their ability to generate smooth surfaces of arbitrary
topology. In this talk we propose a method to generate non-uniform subdivision surfaces interpolating
regular quadrilateral meshes. We show that, choosing a suitable parameterization and properly setting
edge and face point rules, these surfaces compare favorably both with their uniform counterparts and
with non-uniform tensor-product splines.

Potential theory and the equilibrium measure
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Potential theory is a useful tool in Approximation Theory and the equilibrium measure plays a
crucial role. The equilibrium measure is finding its roots in Physics. It is a probability measure which
is minimizing the energy - a double integral with given kernel. In this talk we use the logarithmic and the Riesz kernel and prove some properties of the equilibrium and balayage measures.

Applications of linear barycentric rational interpolation at equispaced nodes
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Efficient linear and infinitely smooth approximation of functions from equidistant samples is a fascinating problem, at least since Runge showed in 1901 that it is not delivered by the interpolating polynomial.

In 1988, I suggested to substitute linear rational for polynomial interpolation by replacing the denominator 1 with a polynomial depending on the nodes, though not on the interpolated function. Unfortunately the so-obtained interpolant converges merely as the square of the mesh size. In 2007, Floater and Hormann have given for every integer a denominator that yields convergence of that prescribed order.

In the present talk I shall present the corresponding interpolant to those not familiar with it, before describing some of its applications, e.g., to differentiation, integration or the solution of boundary value problems. This is joint work with Georges Klein and Michael Floater.

Orthogonal polynomials with respect a quartic complex weight
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We study the large degree asymptotics of monic orthogonal polynomials \( \pi_n(z) \) with the quartic weight \( e^{-Nf(z,t)} \), where \( f(z,t) = 1/2z^2 + 1/4tz^4 \), and \( t \) complex, where the orthogonality is understood as non-Hermitean.

We present the study of the asymptotic for \( t \) in different regions of the complex plane by using a mix of analytic and numerical methods.

In particular near the “gradient catastrophe points” the asymptotic analysis relies upon certain Painlevé transcendents, as known in the literature (Fokas-Its-Kitaev, Duits-Kuijlaars): detailed analysis near the poles of the corresponding transcendent reveals some universal structure.

Classification using nonlinear approximation
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Let \( X \) be a parametric domain that contains the vectors \( x \) of parameters characterizing objects and let \( y = +1 \) for the objects that belong to a particular class and \( y = -1 \) otherwise. For \( Y = \{-1,1\} \) denote by \( \rho \) the underlying probability measure on \( Z = X \times Y \) which factors as \( \rho_X(x) \cdot \rho(y|x) \). If \( p(x) \) is the probability that \( y = +1 \) given \( x \) and \( \eta(x) := E(Y|x) \) is the regression function, then \( \eta(x) = 2p(x) - 1 \).

For given data \( z_i = (x_i, y_i), i = 1, \ldots, n \), drawn independently according to \( \rho \), we want to construct a classifier that predicts the value of \( y \) for any \( x \in X \). It identifies a set \( \Omega \subset X \) of all values for which the prediction is +1. The risk of this classifier is then \( R(\Omega) := \int_X |\chi_\Omega - p|d\rho_X \). The best classifier, called the Bayes classifier, is given by taking \( \Omega = \Omega^* := \{ x : \eta(x) \geq 0 \} \). Its risk \( R(\Omega^*) = \int_X \min\{p, 1-p\}d\rho_X \) is the smallest possible.

A natural way to empirically build a classifier is to consider a finite family \( F \) of sets and choose one of the sets \( \Omega \) from \( F \). A typical approach to characterize the performance of such classifiers is to use smoothness conditions for the boundary of \( \Omega^* \) together with the Tsybakov noise condition. Here we
suggest using instead a special smoothness condition on the function \( \eta \) connected with the sets of the family \( F \).

The talk is based on joint work with Wolfgang Dahmen and Ron DeVore.

**Signal recovery from the roots of the spectrogram**

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First, we review the effect of quasi-periodization on the short-time Fourier transform of band-limited functions. When the window function is a Gaussian and the signal is chosen from a suitable space of signals with finitely many non-zero sample values, the roots of the transform determine each vector up to an overall constant factor. Moreover, recovery from partial information about the roots is possible. If the signal is sufficiently sparse and the quasi-periodization of the short-time Fourier transform is appropriately randomized, then with probability one, knowing part of the roots suffices to recover the vector.

**Characterization of the optimal coefficients of cubature formulas on certain classes of continuous functions**

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For the class \( H^\omega D \) of real-valued functions defined on a convex body \( D \subset \mathbb{R}^d \) and having a given strictly increasing majorant \( \omega \) for the modulus of continuity with respect to an arbitrary norm, we characterize optimal coefficients of formulas of approximate integration with respect to a given finite Borel measure supported on \( D \). This result is used to prove the following statement.

Assume that \( D \) is the convex hull of a node configuration \( X \) in \( \mathbb{R}^d \) equipped with the Euclidean distance, and the triangulation \( \Delta \) of \( X \) in \( D \) is such that at least for one node \( x_i \in X \), the volume of its Voronoi cell does not equal \( \frac{1}{d+1} \) of the total volume of all simplices in \( \Delta \) sharing vertex \( x_i \). Then the piecewise linear interpolating spline corresponding to the triangulation \( \Delta \) is not optimal for the global recovery problem on the class \( H^\omega D \) when the error is measured in the \( L_1 \)-norm.

**On the construction of generators of polyharmonic quasi-int**

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In this talk, we present a simple procedure for the construction of quasi-interpolation operators in spaces of \( m \)-harmonic splines in \( \mathbb{R}^d \), which reproduce polynomials of high degree. The procedure starts from a generator \( \phi_0 \) which is easy to derive but with corresponding quasi-interpolation operator reproducing only linear polynomials and recursively defines generators \( \phi_1, \phi_2, \ldots, \phi_{m-1} \) with corresponding quasi-interpolation operators reproducing polynomials of degree up to \( 3, 5, \ldots, 2m - 1 \) respectively. The construction of \( \phi_j \) from \( \phi_{j-1} \) is explicit, simple and independent of \( m \). We prove that this generator satisfies a refinement equation. The special cases \( d = 2, m = 2, 3 \) are discussed and numerical examples are provided.

**Digital Nets on Spheres**

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Digital Nets as introduced by H. Niederreiter [Monatsh. Math. 104 (1987)] provide a very efficient method to generate sequences of point sets in the \( d \)-dimensional unit cube with desirable properties.
Having low discrepancies, they are well suited for quasi-Monte Carlo rules for approximating high-dimensional integrals over said unit cube.

Such nets can be lifted to the unit sphere in $R^{d+1}$ by means of an area preserving map. Some of the 'good' properties of digital nets should also carry over to the sphere. We present results regarding the discrete energy (that is the Riesz energy, cf. Hardin and Saff [Notices Amer. Math. Soc. 51 (2004), no. 10], and sum of distances), the discrepancy (that is the spherical cap discrepancy, the spherical cap $L_2$-discrepancy and discrepancy with respect to test sets implied by the area preserving mapping) and the worst-case error for numerical integration of such nets on the $d$-sphere.

This talk is based on joint work with Josef Dick and Rob Womersley from UNSW.

**Anisotropic wavelet frames for $L^p$ with irregular translations**

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A very general version of the painless method to construct smooth wavelet frames for $L^2(R^d)$ was recently introduced. These wavelets are compactly supported in frequency and allow for anisotropic dilations and irregular translations.

In this talk we will show the validity of the corresponding atomic decomposition for $L^p(R^d)$, $1 < p < +\infty$.

The results are then used to construct and prove the existence of smooth anisotropic compactly supported (in time) wavelet frames with irregular translations for $L^2(R^d)$ and its corresponding atomic decomposition for $L^p(R^d)$, $1 < p < +\infty$.

We also prove that the atomic decompositions obtained are valid in the family of anisotropic Besov and Triebel-Lizorkin spaces.

**Non-orthogonal Fusion Frames**

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Fusion frames have application to sensor networks, distributed processing and much more. One major problem to date has been that to produce tight fusion frames requires careful placement of the fusion frame subspaces and often in practice we do not have this luxury. We will look at some recent results of Cahill, Casazza and Li using non-orthogonal projections to produce tight fusion frames for most general cases.

**Property enhancement of scalar subdivision schemes**

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In the univariate case, it is known that the mask symbol

$$a(z) = \sum_{\alpha \in Z} a(\alpha) z^{\alpha}, \quad a \in \ell_0(Z), \quad z \in C \setminus \{0\},$$

of any convergent subdivision scheme with dilation 2 is a product of the B-spline mask symbol $\left(\frac{1+z}{2}\right)^k$, $k \in N$, and some other Laurent polynomial [2]. In the scalar multivariate case, the analogous result states that the mask symbol of any convergent subdivision scheme with dilation $2I$ is a shifted affine combination of certain Box spline symbols [1].
We investigate how the properties of bivariate subdivision schemes with Box spline symbols

\[ B_{\alpha,\beta,\gamma}(z_1, z_2) = \left( \frac{1 + z_1}{2} \right)^{\alpha} \left( \frac{1 + z_2}{2} \right)^{\beta} \left( \frac{1 + z_1 z_2}{2} \right)^{\gamma}, \quad \alpha, \beta, \gamma \in \mathbb{N}_0, \]

change, if we combine them in the following way

\[ a(z_1, z_2) = \sum_{(\alpha, \beta, \gamma) \in J} \lambda_{\alpha,\beta,\gamma} \cdot \sigma_{\alpha,\beta,\gamma}(z_1, z_2) \cdot B_{\alpha,\beta,\gamma}(z_1, z_2). \]

The weights \( \lambda_{\alpha,\beta,\gamma} \in \mathbb{R} \) satisfy \( \sum \lambda_{\alpha,\beta,\gamma} = 1 \), the symbols \( \sigma_{\alpha,\beta,\gamma} \) are Laurent polynomials normalized by \( \sigma_{\alpha,\beta,\gamma}(1, 1) = 1 \) and \( J \) is a certain index set.


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**Some Properties of Lebesgue Functions for Polynomial Interpolation at Generalized Chebyshev Nodes**

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To complete the proofs of the optimal nodes for polynomial interpolation given in a previous paper, we must study the deep properties of the corresponding Lebesgue functions from several aspects. In this paper, we first give a formula of the difference of Lebesgue functions in consecutive subintervals. This formula is a generalization of the formulas of the difference of Lebesgue functions in consecutive subintervals for polynomial interpolation at equidistant nodes and Chebyshev nodes. We then extend the monotone properties of the local maxima of Lebesgue functions for polynomial interpolation at Chebyshev nodes, Chebyshev extrema, and equidistant nodes to a large class of sets of nodes. We also give some properties of extrema points of Lebesgue functions on subintervals.

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**Matrix probing and its conditioning**

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When a matrix \( A \) with \( n \) columns is known to be well approximated by a linear combination of basis matrices \( B_1, \ldots, B_p \), we can apply \( A \) to a random vector and solve a linear system to recover this linear combination. The same technique can be used to recover an approximation to \( A^{-1} \). A basic question is whether this linear system is invertible and well-conditioned. In this paper, we show that if the Gram matrix of the \( B_j \)'s is sufficiently well-conditioned and each \( B_j \) has a high numerical rank, then \( n \propto p \log^2 p \) will ensure that the linear system is well-conditioned with high probability. Our main application is probing linear operators with smooth pseudodifferential symbols such as the wave equation Hessian in seismic imaging. We demonstrate numerically that matrix probing can also produce good preconditioners for inverting elliptic operators in variable media.

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**A Thousand Pictures are Worth a Million Words: Mathematical Challenges**

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This presentation is intended for the general audience. No background in advanced mathematics is required to follow at least the first half of the talk.
With the current rapid technological advancement in image data acquisition, the high demand for innovative methods to manipulate and understand large volumes of high-dimensional image data is more urgent than ever. We will be concerned with the manifold approach to image data representation and processing. Understanding of images in terms of spectral curves will be discussed in some detail. We will present an unsupervised data re-organization method, called anisotropic transform, and its integration with random projection for fast computation with arbitrary pre-assigned accuracy in the probability sense. Applications to be discussed include cancerous tissue detection, agricultural control, homeland security, and fast image search. This is a joint work with Jianzhong Wang

Partial parametrization of orthogonal wavelet matrix filters
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In this talk we present some results concerning the (quasi)characterization of orthogonal FIR matrix wavelet filters of full rank type of arbitrary dimension. We propose a parametrization of such filters in terms of orthogonal linear operators acting on vector sequences and with the property that they commute with the shifts by two. In particular we restrict our attention to the explicit construction of a four parameter family of full rank filters of dimension 2 containing the Haar system.

The Grid Refinement Invariance Principle in Functional Data Analysis
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Functional data analysis is an emerging area of statistics wherein the observations are idealized as functions of a continuous variable. In practice one has a finite dimensional approximation of the idealized data, e.g., values of the observed functions on a uniform grid of points in their common domain. As it is often overlooked, it is useful to formulate the principle that as the grid becomes finer, any statistic (a function defined on the data) on the finite dimensional data should converge to an appropriate statistic on the idealized (infinite dimensional) functional data, which typically requires regularization or other methods of approximation. Examples are given to illustrate this principle.

Greedy Space Search
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The so called reduced basis method was introduced for the accurate online evaluation of solutions to a parameter dependent family of partial differential equations which arises, for instance, in the context of optimal control problems. In conventional approaches each parameter evaluation requires the solution of a possibly very large system of equations resulting from a sufficiently accurate finite element discretization. Since for typical instances of such problems the parameter dependent set $F$ of solutions forms a compact set in the underlying “energy space” $H$, the idea is to compute offline a possibly small set of “snapshots” - the reduced basis functions - so that the distance of their linear span from the compact solution set $F$ is smaller than a desired tolerance. Therefore all subsequent frequent online computations require only the solution of small positive definite linear systems of equations. A by now common strategy for computing the reduced basis functions is to employ a greedy strategy. This talk presents some recent results obtained in collaboration with P. Binev, A. Cohen, R. DeVore, G. Petrova, and P. Wojtaszczyk concerning the performance of such a greedy space search in a general
Hilbert space context. Specifically, the achieved accuracy $\sigma_n(\mathcal{F})$ after $n$ greedy steps is compared with the best possible accuracy given by the Kolmogorov $n$-widths $d_n(\mathcal{F})_H$ of $\mathcal{F}$. We indicate the main concepts needed to show, for instance, that whenever the Kolmogorov $n$--widths decay at any polynomial rate, then so do the greedy errors, while, in general, the best possible bound for a direct comparison of $\sigma_n(\mathcal{F})$ and $d_n(\mathcal{F})_H$ is of the form $\sigma_n(\mathcal{F}) \leq C2^nd_n(\mathcal{F})_H$.

A new approach to data matching and alignment

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In this talk we will introduce a new approach to data matching and alignment which we have developed with Charles Fefferman at Princeton.

The idea can be described as follows. It is a classical result in Euclidean Geometry that given two sets of data points in Euclidean $d$ Space whose pairwise distances are equal, there exists an isometry that maps the one set of points 1-1 and onto the other. See for example the treatise of Praslov and Tikhomirov.

In order to deal with questions for example of noise in data analysis, robotics, computer vision for example, one often needs to ask the same question but demand that instead of the pairwise distances to be equal, they should be "close" in some reasonable metric.

Given such an assumption, can such an isometry be found. We look at the case of rigid $\varepsilon$ distortion maps, $\varepsilon$ distorted diffeomorphisms and in three joint papers solve the problem completely. We have positive and negative results which we are now applying to various problems.

References: See http://math.georgiasouthern.edu/~damelin

Optimal Assembly of System Matrices in Bernstein-Bézier Form

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Numerical solution of elliptic partial differential equations discretized in weak form by finite elements or multivariate splines requires computation of the mass and stiffness matrices. This step becomes expensive if high order elements are used. We show that Bernstein-Bézier representation is particularly advantageous for the assembly of these matrices because it allows to perform it with optimal complexity $O(n^{2d})$ on each simplex, where $d$ is the number of variables and $n$ is the polynomial degree. The results are obtained jointly with M. Ainsworth and G. Andriamaro.

Asymptotics for Christoffel functions based on orthogonal rational functions

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Suppose the rational functions $\{\varphi_j\}$, with arbitrary complex poles bounded away from the interval $I = [-1, 1]$, form an orthonormal system with respect to a positive bounded Borel measure $\mu$ on $I$, and let the associated rational Christoffel functions be given by $\lambda_n(x) = \frac{1}{\left(\sum_{j=0}^{n-1} |\varphi_j(x)|^2\right)^{-1}}$. We then prove convergence of the sequence $\{n\lambda_n(x)\}_{n>0}$, for $x \in I$ and $n$ tending to infinity, under the assumption that the measure $\mu$ is regular in the sense of Stahl and Totik, absolutely continuous in an open interval containing $x$, and positive and continuous in a neighborhood of $x$. 
A sequence of linear operators \((L_n)\) from one Banach space \(X\) into another is said to converge \textbf{arbitrarily slowly} (resp., \textbf{almost arbitrarily slowly}) to a linear operator \(L\) provided that: (1) \((L_n)\) converges to \(L\) pointwise (i.e., \(|L_n(x) - L(x)| \to 0\) for each \(x\)), and (2) For each sequence of real numbers \((\epsilon_n)\) converging to 0, there exists \(x \in X\) such that \(|L_n(x) - L(x)| \geq \epsilon_n\) for all \(n\) (resp., for infinitely many \(n\)).

Almost arbitrarily slow convergence is characterized and a useful sufficient condition for arbitrarily slow convergence is established. Applications include: The Bernstein polynomial operators converge arbitrarily slowly to the identity operator. The Hermite-Fejer, the Landau, the Fejer, and the Jackson operators all converge almost arbitrarily slowly to the identity operator. All the classical quadrature rules (e.g., the compound trapezoidal and Simpson’s rules, as well as Gaussian quadrature) all converge almost arbitrarily slowly to the definite integral functional. The von Neumann-Halperin method of cyclic projections satisfies a dichotomy theorem: either it converges linearly (i.e., “fast”) or it converges arbitrarily slowly. No intermediate type of convergence is possible. Whichever way it converges is determined by noting whether a certain sum of orthogonal subspaces is closed or not.

\[ 1 + \phi(d) \] is the largest positive zero of a certain sequence of monic polynomials of degree \(2d - 1\) with integer coefficients which we call Gonchar polynomials. Rather surprisingly, \(\phi(2)\) is the Golden ratio and \(\phi(4)\) the lesser known Plastic number. But Gonchar polynomials have other interesting properties. We discuss their factorizations, investigate their zeros and present some challenging conjectures.

The talk presents approximation methods for set-valued functions, based on adaptation of sample-based approximation operators for real-valued functions. The adaptation is done by replacing operations between numbers by operations between sets. First we discuss approximation methods in the case that the error is measured in the Hausdorff metric, and give error estimates in terms of the regularity properties of the approximated set-valued function. Then we present the interpolatory 4-point subdivision scheme, adapted to sets, using a new binary average between sets, designed for the case that the approximation error is measured in the symmetric-difference metric. The application of the latter method to the reconstruction of 3D objects from parallel 2D cross-sections is demonstrated by examples, which indicate the quality of this method. The talk reports on joint works with E. Farkhi, A. Mokhov and S. Kels.
Uncertainty principles on Riemannian manifolds  
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In this talk, we derive an uncertainty principle for general Riemannian manifolds M. Similar as for the classical Heisenberg principle on the real line or the Breitenberger principle on the unit circle, the proof of the uncertainty rests upon an operator theoretic approach involving the commutator of two operators on a Hilbert space. As a frequency operator, we will use a particular differential-difference operator, a so called Dunkl operator, which plays the role of a generalized root of the radial part of the Laplace-Beltrami operator on M. Subsequently, we will show with a family of Gaussian-like functions that the deduced uncertainty inequality on the Riemannian manifold is in fact sharp. Finally, we specify in more detail the uncertainty principles for well-known Riemannian manifolds like the d-dimensional unit sphere, the hyperbolic spaces and the projective spaces.

Particularly interesting in these uncertainty relations is the formula for the space variance. For the unit sphere and for the projective spaces, we are able to find those harmonic polynomials of a fixed degree n that minimize the term for the space variance, i.e. those polynomials that are best localized in space with respect to the given uncertainty principle.

Stable Evaluation of Gaussian RBF Interpolants  
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We present a new way to compute and evaluate Gaussian radial basis function interpolants in a stable way also for small values of the shape parameter, i.e., for “flat” kernels. This work is motivated by the fundamental ideas proposed earlier by Bengt Fornberg and his co-workers. However, following Mercer’s theorem, an $L_2(\mathbb{R}^d)$-orthonormal expansion of the Gaussian kernel allows us to come up with an algorithm that is simpler than the one proposed by Fornberg, Larsson and Flyer and that is applicable in arbitrary space dimensions $d$.

Interpolatory subdivision of irregularly spaced data  
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I will talk about recent work on establishing the Holder regularity and approximation properties of interpolatory subdivision schemes in the case that the data are irregularly spaced. The focus will be on the generalization of the family of schemes based on polynomials of odd degree proposed by Dubuc and Deslauriers.

Hard Thresholding Pursuit:  
an algorithm for Compressive Sensing  
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We introduce a new iterative algorithm to find sparse solutions of underdetermined linear systems. The algorithm, a simple combination of the Iterative Hard Thresholding algorithm and of the Compressive Sampling Matching Pursuit algorithm, is called Hard Thresholding Pursuit. We study its general convergence, and notice in particular that only a finite number of iterations are required. We then
show that, under a certain condition on the restricted isometry constant of the matrix of the linear system, the Hard Thresholding Pursuit algorithm indeed finds all $s$-sparse solutions. This condition, which reads $\delta_{3s} < 1/\sqrt{3}$, is heuristically better than the sufficient conditions currently available for other Compressive Sensing algorithms. It applies to fast versions of the algorithm, too, including the Iterative Hard Thresholding algorithm. Stability with respect to sparsity defect and robustness with respect to measurement error are also guaranteed under the condition $\delta_{3s} < 1/\sqrt{3}$. We conclude with some numerical experiments to demonstrate the good empirical performance and the low complexity of the Hard Thresholding Pursuit algorithm.

The dimension of trivariate spline spaces on Alfeld splits
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TBA

Wavelet inversion formulae associated to abelian matrix groups
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Continuous wavelet inversion formulae are closely related to a generalized Calderon condition. We first present a characterization of the matrix groups in dimensions $> 1$ that admit square-integrable functions satisfying such a condition. We then focus on the subclass of abelian matrix groups, described by finitely many (infinitesimal) generators. If all group elements have positive spectrum, the existence of a wavelet inversion formula can be verified by standard linear algebra techniques. However, in general the simple criteria valid for the positive spectrum case do not apply.

Interpolation on Embedded Submanifolds with RBFs: Sobolev Error Estimates
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In this talk we investigate the approximation properties of kernel interpolants on manifolds. The kernels we consider will be obtained by the restriction of positive definite kernels on $\mathbb{R}^d$, such as radial basis functions (RBFs), to a smooth, compact embedded submanifold $M \subset \mathbb{R}^d$. For restricted kernels having finite smoothness, we provide a complete characterization of the native space on $M$. After this and some preliminary setup, we present Sobolev-type error estimates for scattered data interpolation problems.

Some Intermediate Spaces between $L_p$ and $L_\infty$
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We introduce new intermediate functional spaces between $L_p$ and $L_\infty$ and study their properties.
Some Properties of Uniform B-splines and Box Splines

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We study refinable properties of uniform B-splines and in particular their use for constructing wavelet frames, which is then extended to box splines. We also consider how uniform B-splines approximate the Gaussian: this has connections to Heisenberg’s uncertainty principle and scale-space operators with causality properties, as used in computer vision.

Quasi-uniformity of Minimal Weighted Energy Points on Compac

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The problem of finding configurations of points that are optimally-distributed on a set appears in a number of guises including best-packing problems, coding theory, geometrical modeling, statistical sampling, radial basis approximation and golf-ball design (i.e., where to put the dimples). We consider geometrical properties of N-point configurations \( \{x_i\}_{i=1}^N \) on a compact metric set \( A \) (with metric \( m \)) that minimize a weighted Riesz \( s \)-energy functional of the form

\[
\sum_{i \neq j} \frac{w(x_i, x_j)}{m(x_i, x_j)^s},
\]

for a given ‘weight’ function \( w \) on \( A \times A \) and a parameter \( s > 0 \).

Specifically, if \( A \) supports an (Ahlfors) \( \alpha \)-regular measure \( \mu \), we prove that whenever \( s > \alpha \), any sequence of weighted minimal Riesz \( s \)-energy \( N \)-point configurations on \( A \) (for ‘nice’ weights) is quasi-uniform in the sense that the ratios of its mesh norm to separation distance remain bounded as \( N \) grows large. Furthermore, if \( A \) is an \( \alpha \)-rectifiable compact subset of Euclidean space with positive and finite \( \alpha \)-dimensional Hausdorff measure, one may choose the weight \( w \) to generate a quasi-uniform sequence of configurations that also has (as \( N \to \infty \)) a prescribed positive continuous limit distribution with respect to \( \alpha \)-dimensional Hausdorff measure. This is joint work with E. Saff and T. Whitehouse.

Normal multi-scale transform for curves based on combining several subdivision schemes

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Normal multi-scale transforms (MTs) allow for the efficient computational processing of densily sampled (or analytically given) geometric objects of co-dimension one. E.g., given a curve \( C \) in \( R^2 \), an initial sequence of vertices \( V^0 \subset C \) (creating a polygonal line interpolating \( C \)), and a univariate subdivision operator \( S \), a normal MT produces denser vertex sets \( V^j \subset C \) and polygonal lines associated with them according to

\[
V^j = SV^{j-1} + d^j n^j,
\]

where \( n^j \) is a set of unit ”normals” that can be computed solely from \( V^{j-1} \), and \( d^j \) is the scalar ”detail” sequence of the signed distances between the prediction points \( SV^{j-1} \) and the corresponding imputation points \( V^j \). Normal MTs based on interpolating operators \( S \) have been analyzed in [1], while normal MTs based on approximating operators \( S \) have been analyzed in [2]. There is always a tradeoff between global well-posedness of the process (i.e., guaranteed existence of admissible imputation points at all levels) on the one side, and fast detail decay rate, together with smooth normal re-parameterization, on the other. The aim of this talk is to propose new normal MTs that are globally well-defined on convex/concave data, and still possess high detail decay rate and regularity of the normal re-parameterization. The main
idea in our construction is to combine the "data-uniformizing" properties of the B-spline subdivision operators with the "detail-improving" properties of the Deslauriers-Dubuc subdivision operators. In the end, an extension to geometrical subdivision operator $S$ will also be discussed.


Wavelet Tight Frames for Linear NURBS

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In order to extend the theory of wavelet tight frames on B-splines to NURBS, we start with the linear NURBS. First we answer the question: for the given weights and knot sequence, if the compactly supported wavelet tight frames for linear NURBS always exist. Then we provide explicit formulas for wavelet coefficients. Finally, we study the properties of linear NURBS tight frames by comparing the differences between B-spline MRA tight frames and NURBS MRA tight frames.

Chromatic Derivatives and Approximations

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Chromatic derivatives are special, numerically robust linear differential operators which provide a unification of a broad class of families of orthogonal polynomials with a broad class of special functions. They are used to define chromatic expansions, which provide a framework for local approximation of analytic functions, generalizing the Neumann series of Bessel functions. Chromatic expansions are motivated by signal processing, because they provide local signal representation, akin to the Taylor expansion. They are complementary to the global signal representation provided by the Shannon sampling expansion, but, unlike the Taylor expansion, they share all properties of the Shannon expansion which are crucial for signal processing. We will try to demonstrate that chromatic derivatives and expansions not only have rich and intriguing mathematical properties, but are also new, promising tools for applications.

The discrete support of subdivision

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The support of a subdivision scheme is the region of the subdivision curve or surface which is affected by the displacement of a single control point. Assuming a continuous geometry, the support of curve subdivision schemes and the regular case of the major surface subdivision schemes is well understood. However, in practice, the implemented subdivision curves and surfaces have a discrete geometry almost always, that is, the control points at any subdivision level lie on the vertices of a fine grid introduced by the finite precision of the coordinates. In this talk, we study the discrete support of curve and surface subdivision schemes assuming, for simplicity, that all new control points are computed with infinite precision and then clipped to the nearest vertex of the grid. This discrete support, which is always a subset of the continuous support, is a function the magnitude of the displacement of the control point which in some simple cases can be easily computed explicitly. In curve subdivision in particular, the discrete support is related to the finite number of sign changes of the basis function and thus, related to the visual quality of the subdivision curve.
Geometric Hermite interpolation by rational Bézier spatial curves
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Geometric interpolation by parametric polynomial curves became one of the standard techniques for interpolation of geometric data. A natural generalization leads to rational geometric interpolation schemes. The aim of this talk is to present a general framework for Hermite geometric interpolation by rational Bézier spatial curves. In particular, the cubic $G_2$ and quartic $G_3$ interpolation are analyzed in detail. Systems of nonlinear equations are derived and the analysis of the existence of admissible solutions is studied. For the cubic case the solution is obtained in a closed form and geometric conditions on its existence are given. The quartic case transforms into solving a univariate quartic equation. The asymptotic analysis is done.

On positivity of principal minors of bivariate Bezier collocation matrix
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It is well known that the bivariate polynomial interpolation problem at domain points of a triangle is correct. Thus the corresponding interpolation matrix M is nonsingular. L.L. Schumaker stated the conjecture, that the determinant of M is positive. Furthermore, all its principal minors are conjectured to be positive, too. This result would solve the constrained interpolation problem.

In this talk, the basic conjecture for the matrix M, the conjecture on minors of polynomials for degree $17$ and for some particular configurations of domain points are confirmed. Some interesting hypotheses, including a precise lower bound for minors of M and interlacing of eigenvalues will be stated.

Kolmogorov problem for $d$ numbers on the class of multiply monotone functions
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Let $G$ be real line $\mathbb{R}$ or nonpositive halfline $\mathbb{R}_-$. By $L_\infty(G)$ we will denote the space of all measurable essentially bounded functions $x: G \to \mathbb{R}$ with usual norm $\|x\| = \|x\|_{L_\infty(G)}$. For $r \in \mathbb{N}$ we will denote by $L^r_\infty(G)$ the space of all functions $x: G \to \mathbb{R}$ that have derivatives up to and including the order $r - 1$ (in the case $G = \mathbb{R}_-$ we take, as usual, the one-sided derivative at the point $t = 0$) such that derivatives $x^{(r-1)}$ are locally absolutely continuous and $x^{(r)} \in L_\infty(G)$. Define $L^r_{\infty,\infty}(G) := L^r_\infty(G) \cap L_\infty(G)$.

Kolmogorov formulated the following problem. Let some class $X \subset L^r_{\infty,\infty}(G)$ and the arbitrary system of integers

$$0 \leq k_0 < k_1 < \ldots < k_d = r$$

be given. The problem is to find necessary and sufficient conditions for the system of positive numbers

$$M_{k_0}, \ldots, M_{k_d}$$

to guarantee the existence of the function $x \in X$, such that

$$\|x^{(k_i)}\| = M_{k_i}, \ i = 0, \ldots, d.$$

In this talk we shall present a solution of Kolmogorov problem (for any $d$) on the class of multiply monotone functions $X = L^r_{\infty,\infty}(\mathbb{R}_-)$, which is the class of functions $x \in L^r_{\infty,\infty}(\mathbb{R}_-)$ that are nonnegative.
along with all their derivatives up to and including order $r$ (derivative of order $r$ must be nonnegative almost everywhere).

**Frames and Wiener’s Tauberian Lemma**

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I will discuss different non-commutative extensions of Wiener’s lemma with relation to localization of different kinds of frames and their duals.

**Data Separation by Sparse Approximation**

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Along with the deluge of data we face today, it is not surprising that the complexity of such data is also increasing. One instance of this phenomenon is the occurrence of multiple components, and hence, analyzing such data typically involves a separation step. One most intriguing example comes from neurobiological imaging, where images of neurons from Alzheimer infected brains are studied with the hope to detect specific artifacts of this disease. The prominent parts of images of neurons are spines (pointlike structures) and dendrites (curvelike structures), which require separate analyzes, for instance, counting the number of spines of a particular shape, and determining the thickness of dendrites.

In this talk, we will first introduce a general methodology for separating morphologically distinct components using ideas from sparse approximation. More precisely, this methodology utilizes two representation systems each providing sparse approximations of one of the components; the separation is then performed by thresholding. After introducing this method, we provide an estimate for its accuracy. We then study this separation approach using a pair of wavelets (adapted to pointlike structures) and shearlets (adapted to curvelike structures) for separating spines and dendrites. Finally, we discuss details of the implementation and present numerical examples to illustrate the performance of our methodology.

This talk contains joint work with David Donoho (Stanford University) and Wang-Q Lim (University of Osnabrueck).

**Multiscale analysis in Sobolev spaces by scaled radial basis functions on the unit sphere**

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In the geosciences, data are often collected at scattered sites on the unit sphere. Moreover, the geophysical data typically occur at many length scales: for example, the topography of Central Australia varies slowly, while that of the Himalayan mountains varies rapidly. To handle multiscale data at scattered locations, we consider an approximation scheme based on radial basis functions of different scales, which are generated from a single underlying radial basis function (RBF) $\Phi$ using a sequence of scales $\delta_1, \delta_2, \ldots$ with limit zero. The scaled RBF is defined by $\Phi_\delta = c_\delta \Phi (\frac{x}{\delta})$, which is then restricted to the unit sphere. A multiscale approximation method using scaled radial basis functions was proposed and analysed in [1].

In the present work, we prove convergence results for target functions outside the reproducing kernel Hilbert space of the employed kernel. We also discuss the sparsity of the coefficients that are used to represent the approximations, leading to a new compression technique for functions on the sphere. Numerical experiments using topographical data of the Earth will be presented.
Shearlets and sparse approximations in $L^2(R^3)$
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We introduce a class of generalized three-dimensional cartoon-like images, i.e., functions of three variables which are piecewise $C^\beta$ smooth with discontinuities on a $C^\alpha$ surface for $1 < \alpha \leq \beta \leq 2$, and consider sparse, nonlinear approximations of such. The optimal rate of approximation which is achievable within the class of functions will be derived. We then introduce three-dimensional pyramid-adapted shearlet systems with compactly supported generators and prove that such shearlet systems indeed deliver nearly optimal sparse approximations of three-dimensional cartoon-like images. Finally, we extend this result to the situation of piecewise $C^\alpha$ discontinuity surfaces, and again derive nearly optimal sparsity of the constructed shearlet frames.

Modeling Data by Multiple Subspaces: Theory and Algorithms
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We study the problem of modeling data by several affine subspaces, which generalizes the common modeling by a single subspace. It arises, for example, in object tracking and structure from motion. One of the simplest methods for such modeling is based on energy minimization, where the energy involves $p$th powers of distances of data points from subspaces. We first review some theory by Lerman and Zhang of the effectiveness of such energy minimization for recovering all subspaces simultaneously and also recovering the most significant subspace. We reveal the following phase transition in a special setting (e.g. spherically symmetric outliers): when $p=1$ the underlying subspaces can be recovered by such energy minimization; whereas when $p>1$ the underlying subspaces are sufficiently far from the global minimizer. Nevertheless, for more general settings (i.e., outliers which are not spherically symmetric) we can point at some disadvantages of the energy minimization strategy. In order to practically solve the problem, we present a simple and fast geometric method for multiple subspaces modeling. It forms a collection of local best fit affine subspaces, where the size of the local neighborhoods is determined automatically by the Peter Jones beta numbers. This collection of subspaces can then be further processed in various ways. For example, greedy selection procedure according to an appropriate energy or a spectral method to generate the final model. We demonstrate the state of the art accuracy and speed of the suggested procedure on applications for several applications.

Approximation of some classes of multivariate functions by harmonic splines
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For univariate functions from classes $W^2_2[a, b]$ and $W^2_p[a, b]$ there are well known exact estimates of error of interpolation by piecewise-linear functions in $L_p([a, b])$-norm, $1 \leq p \leq \infty$, and in $L_1([a, b])$-norm, respectively.

One can consider piecewise-harmonic functions (harmonic splines) as a natural multidimensional generalization of piecewise-linear functions. Therefore we obtained the exact value of upper bound of approximation error by harmonic splines for functions $u$ defined on parallelepiped $\Omega \in R^n$ such that $\|\Delta u\|_{L_\infty(\Omega)} \leq 1$ as well as for functions $u$ such that $\|\Delta u\|_{L_p(\Omega)} \leq 1$, $1 \leq p \leq \infty$. In first case error is found in $L_p(\Omega)$-norm, $1 \leq p \leq \infty$, and in second case in $L_1(\Omega)$-norm.
**L-approximation to the Gaussian**
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This talk considers best $L^1$-approximation and best one-sided approximation by entire functions of exponential type $\tau$.

The functions to be approximated include the Gaussian $G_\lambda(x) = e^{-\lambda x^2}$ ($\lambda > 0$) and the truncated Gaussian which equals $G_\lambda(x)$ for positive $x$ and is identically zero for negative $x$. The functions of best approximation have explicit representations involving Laplace transforms of Jacobi-theta functions and their truncations. The motivation to study these came from a problem in Number Theory. This talk describes the extremal functions and how they are applied. (Joint work with Emanuel Carneiro and Jeff Vaaler.)

**Gigapixel Binary Sensing**
*Image Acquisition Using One-Bit Poisson Statistics*
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Before the advent of digital image sensors, photography, for the most part of its history, used film to record light information. In this talk, I will present a new digital image sensor that is reminiscent of photographic film. Each pixel in the sensor has a binary response, giving only a one-bit quantized measurement of the local light intensity.

To analyze its performance, we formulate the binary sensing scheme as a parameter estimation problem based on quantized Poisson statistics. We show that, with a single-photon quantization threshold and large oversampling factors, the Cramer-Rao lower bound of the estimation variance approaches that of an ideal unquantized sensor, that is, as if there were no quantization in the sensor measurements. Furthermore, this theoretical performance bound is shown to be asymptotically achievable by practical image reconstruction algorithms based on maximum likelihood estimators.

Numerical results on both synthetic data and images taken by a prototype sensor verify the theoretical analysis and the effectiveness of the proposed image reconstruction algorithm. They also demonstrate the benefit of using the new binary sensor in high dynamic range photography.

**Universality Limits In Random Matrices and Orthogonal Polynomials**
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It is a remarkable fact that universality limits in the theory of random matrices, can be studied using classical aspects from the theory of orthogonal polynomials. We shall start by discussing how random matrices were used by the physicist Eugene Wigner to model interactions amongst nuclei, and then outline the connection (established by Freeman Dyson, M. Mehta, and others) to orthogonal polynomials. We shall survey some recent techniques for investigating just how universal is that universality.

**Simplex Splines for the Powell-Sabin 12 Split**
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A Simplex Spline is the natural generalization of a B-spline to several dimensions. We give a short introduction to simplex splines and some simplex spline spaces and then introduce a simplex spline
basis for a space of C1 quadratics on the well-known Powell-Sabin 12-split triangular region. This basis has many desirable properties and can be used to approximate C\(^1\)-surfaces on arbitrary triangulations.

Solid subdivision schemes revisited for isogeometric analysis
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Isogeometric analysis is an active research area in directly using CAD representations for integrated design and analysis. The most commonly used representations for isogeometric analysis are NURBS and T-splines at the moment. This talk presents some ongoing exploration on subdivision solid modelling for isogeometric analysis. Several solid subdivision schemes will be revisited. Major operations and challenges towards isogeometric analysis using subdivision solids will be discussed. Related issues on subdivision solid model construction from either a known boundary representation model or other volumetric data will also be addressed.

Multiscale geometric methods for high-dimensional data sets
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We discuss techniques for the geometric multiscale analysis of intrinsically low-dimensional point clouds. We first show how such techniques may be used to estimate the intrinsic dimension of data sets, then discuss a novel geometric multiscale transform, based on what we call geometric wavelets, that leads to novel approximation schemes for point clouds, and dictionary learning methods for data sets. Finally, we apply similar techniques to model estimation when points are sampled from a measure supported on a union of an unknown number of unknown planes of unknown dimensions.

D.Newman problem on the arcs in the complex plane in the integral metric
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Recall that the D.Newman problem is connected with the Jackson theorem for non-periodic functions on the segment \([-1, 1]\), i.e.

\[
f \in \text{Lip}\([-1,1]\] \Rightarrow E_n(f; [-1, 1]) \leq \frac{\text{const}}{n}
\]

the problem that states: what necessary and sufficient conditions should satisfy the arc in the complex plane in order the Jackson theorem be valid on it, was solved in J.I.Mamedhanov and V.V.Maymeskul’s papers.

In our option, this problem remains actual in an integral metric as well. The more so, up to present, there is in constructive characteristics for the Lipschitz class of order \(\alpha\) on the segment \([-1, 1]\) in the metric \(L_p\), i.e. there is no analogue of Nikolskii-Timan-Dzyadyk theorem in the metric \(L_p[-1, 1]\).

In this report we consider the arcs \(\Gamma\) determined by the parametric equation \(z = z(s), (0 \leq s \leq \ell, \ell\) is the length of the arc \(\Gamma\)) and introduce the class \(H^\alpha_p(\Gamma) (0 < \alpha \leq 1)\) (Lipschitz-Hölder class) determined in the following way

\[
\omega_p(f, \delta)_{\Gamma} = \sup_{|h| \leq \delta} \left( \int_0^b |f(z(s + h)) - f(z(s))|^p |z'(s)| ds \right)^{1/p} \leq \text{const}\delta^\alpha.
\]
The following theorem is valid

**Theorem.** If $\Gamma$ is a smooth arc and $f \in H^\alpha_p(\Gamma)$ ($0 < \alpha \leq 1$), then

$$\rho_n(f, \Gamma) = \inf_{P_n} \|f - P_n\|_{L_p(\Gamma)} \leq constn^{-\alpha}.$$ 

**Besov spaces on Carnot groups: Yet another example of coorbit spaces**

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In this talk we shall present the characterization of homogeneous Besov spaces on Carnot (stratiﬁed) groups in terms of smooth and frequency localized continuous wavelets. As an application of this characterization we cover these spaces by the theory of J. Christensen and G. Ólafsson and describe them as generalized coorbit spaces.

This talk is based on joint works with Jens Christensen (University of Maryland), Gestur Ólafsson (Louisiana State University), and Hartmut Führ (RWTH-Aachen University).

**An inequality for bi-orthogonal pairs.**  
*After Rademacher, Menshov, and Salem.*  
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Ideas from Salem’s 1941 proof of the Rademacher-Menshov Theorem can be used to prove an inequality of Kwapien and Pełczyński concerning a lower bound for partial sums of series of bi-orthogonal vectors in a Hilbert space, or the dual vectors. This is applied to some lower bounds on Lebesgue constants for some orthogonal expansions.

**Minimum Sobolev norm interpolation with trigonometric polynomials**  
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Let $q \geq 1$ be an integer, $y_1, \cdots, y_M \in [-\pi, \pi]^q$, and $\eta$ be the minimal separation among these points. Given the samples $\{f(y_j)\}_{j=1}^M$ of a smooth target function $f$ of $q$ variables, $2\pi$–periodic in each variable, we consider the problem of constructing a $q$–variate trigonometric polynomial of spherical degree $O(\eta^{-1})$ which interpolates the given data, remains bounded in the Sobolev norm (independent of $\eta$ or $M$) on $[-\pi, \pi]^q$, and converges to the function $f$ on the set where the data becomes dense. We prove that the solution of an appropriate optimization problem leads to such an interpolant. Numerical examples are given to demonstrate that this procedure overcomes the Runge phenomenon when interpolation at equidistant nodes on $[-1, 1]$ is constructed, and also provides a respectable approximation for bivariate grid data, which does not become dense on the whole domain.

**Dimension Reduction in Sparsity and Sampling**  
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Given a set of points $F$ in a high dimensional space, the problem of finding a union of subspaces $\bigcup_i V_i \subset \mathbb{R}^N$ that best explains the data $F$ increases dramatically with the dimension of $\mathbb{R}^N$. In this
article, we study a class of transformations that map the problem into another one in lower dimension. We use the best model in the low dimensional space to approximate the best solution in the original high dimensional space. We then estimate the error produced between this solution and the optimal solution in the high dimensional space.

\[ L_p \] Sobolev estimates for functions vanishing on a Lipschitz domain on a compact manifold

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Essential to many estimates in scattered data approximation are \( L_p \) Sobolev-space estimates on functions having zeros quasi-uniformly distributed on a Lipschitz domain \( \Omega \). Such estimates are known for \( \Omega \subset \mathbb{R}^d \). Results for \( \Omega \subset M \), where \( M \) is \( C^\infty \) compact Riemannian manifold \( M \), are a different matter. Because of the non-Euclidean metric on \( M \), the manifold is very different from the Euclidean one. Recently, Thomas Hangelbroek, Joe Ward and I obtained such estimates for \( \Omega \subset M \). The proofs involved make use of many geometric ideas. In the end, the results for the manifold case turn out to be intrinsic, and to hold in the same generality as those in the Euclidean case. The bounds and the conditions needed reflect geometric properties of \( \partial \Omega \) and \( M \), but are independent of the volume and diameter of \( \Omega \).

Compressed Sensing with Coherent and Redundant Dictionaries
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This talk will present new results on signal recovery from undersampled data for which the signal is not sparse in an orthonormal basis, but rather in some arbitrary dictionary. The dictionary need not even be incoherent, and thus this work bridges a gap in the literature by showing that signal recovery is feasible for truly redundant dictionaries. We show that the recovery can be accomplished by solving an \( L_1 \)-analysis optimization problem. The condition on the measurement matrix required for recovery is a natural generalization of the well-known restricted isometry property (RIP), and we show that many classes of matrices satisfy this property. This condition does not impose any incoherence requirement on the dictionary, so our results hold for dictionaries which may be highly overcomplete and coherent. We will also show numerical results which highlight the potential of the \( L_1 \)-analysis problem.

Sampling on Commutative Spaces
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Several interesting (and useful) spaces \( X \) can be realized as commutative spaces, that is \( X = G/K \) where \( G \) is a connected Lie group and \( K \) is a compact subgroup such that the Banach algebra \( L^1(G/K) \) is commutative. This include the simple example \( X = \mathbb{R}^d \) as well as the unite disc with the Poincare metric. We give an overview of basic harmonic analysis on those spaces and discuss resent sampling result on spaces of bandlimited functions.

The Effect of Filtering Directional Bias on Analog to Digital Conversion in Multidimensions
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filters for 2D or 3D-images are generated as the Fourier transforms of square-integrable $Z^d$-periodic functions ($d = 2, 3$). Depending on the variability of the decay rate of the filter’s Fourier transform, which we call directional bias of the filter, the reconstruction errors of an input image $f$ may vary. This variation also depends on the rotations of the input image. More specifically, we study the effects on the truncation error $E_N(f) = \inf \{ \| f - \sigma_I(f) \| : I \subset Z^d | I \leq N \}$ of the directional distribution of the decay of $\hat{\psi}$ and $\phi$, where $\sigma_I(f)(x) = \sum_{n \in I} < f, \psi(-n) > \phi(x-n)$, $\phi$ is the reconstruction kernel and $\psi$ is the analysis kernel. We demonstrate the effects of the directional distribution of the decays of the analysis and synthesis kernels on the construction of artifact-free synthetic dendritic arbors used as phantoms for benchmarking the accuracy of neuroscience imaging software. These synthetic data are part of joint work with P.H. Herrera and I.A. Kakadiaris.

**Exact Inequalities of Kolmogorov Type for Fractional Derivatives of Multivariate Functions**

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Let $L^\infty_{\alpha, \infty}(R^m)$ be the spaces of functions $f \in L^\infty_{\alpha}(R^m)$, such that $\Delta f \in L^\infty_{\infty}(R^m)$. New exact inequalities of Kolmogorov type that estimate uniform norm of Riesz derivative of a function $f \in L^\infty_{\alpha, \infty}(R^m)$ with the help of uniform norm of $f$ and uniform norm of $\Delta f$ are obtained.

The Stechkin problem about approximation of unbounded operators $D^\alpha$ by bounded ones on the class of functions $f \in L^\infty_{\alpha, \infty}(R^m)$ such that $\| \Delta f \|_{\infty} \leq 1$ and the problem about the best recovery of the operator $D^\alpha$ on the elements of this class, given with the error $\delta$ are solved.

**Splines on manifolds and the spherical Radon transform**

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We extend the classical theory of variational interpolating splines to the case of compact Riemannian manifolds. Our consideration includes in particular such problems as interpolation of a function by its values on a discrete set of points and interpolation by values of integrals over a family of submanifolds.

The existence and uniqueness of interpolating variational splines on a Riemannian manifold is proven. It is shown that variational splines are polyharmonic functions with singularities. Optimal properties of such splines are shown. The explicit formulas of variational splines in terms of the eigenfunctions of Laplace-Beltrami operator are found. It is also shown that in the case of interpolation on discrete sets of points variational splines converge to a function in $C^k$ norms on manifolds.

Applications of these results to the hemispherical and Radon transforms on the unit sphere are given.

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**$C^\infty$ compactly supported radial basis function approximations**

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Approximations based on analytic radial basis functions (RBFs), such as Gaussians and multiquadrics, are often unstable when scattered nodes are used. In particular, analytic basis functions lead to Lebesgue constants that grow exponentially with the number of nodes if interpolation is carried out.
on arbitrary point sets on complex domains. In this talk we explore convergence and stability properties of $C^\infty$ compactly supported RBF interpolation. By compromising geometric convergence, better conditioned schemes with sparser matrices can be constructed.

On Approximation on Discrete Sets
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We present results on polynomial and rational approximation of analytic functions on discrete sets of points in the complex plane. Some questions related to the discrete Hankel operator will be discussed.

Hyperinterpolation and discrete polynomial approximators on Euclidean embedded homogeneous manifolds.
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Each sphere or projective space is a compact connected weakly symmetric space $M$, with metric $\rho$, which can be isometrically embedded in an Euclidean space, $E$, with isometry group $G = G(M) \subset O(E)$, the orthogonal group, and carries a unique normalized $G$-fixed measure $\mu$. Most standard Banach spaces of functions or measures on $M$ admit a (left) $G$ action which leaves the norm unchanged and for which the polynomials or polynomials times $\mu$ are dense or weak*-dense. Each member of the family $\{P_n\}$ of polynomials of degree at most $n$ restricted to $M$ is $G$-invariant and finite dimensional. This family leads to an extensive theory of constructive polynomial approximation on $M$.

A positive weight quadrature rule on $M$ is any discrete probability measure $\nu_{A^n,Y_n} = \sum_{y \in Y_n} A^n_y \delta_y \in \mathcal{M}(M)$. When $\int pd\nu_{A^n,Y_n} = \int pd\nu$, $\forall \nu \in \mathcal{P}_{2n}$ and $\kappa_n(x,y)$ is the ($G$-invariant) reproducing kernel for $P_n \subset L^2(\mu)$ the hyperinterpolation operator $L_n : C(M) \to \mathcal{P}_n$ defined by:

$$L_n(f)(x) := \nu_{A^n,Y_n}(f(\cdot)\kappa_n(x,\cdot)) = \sum_{y \in Y_n} f(y)\kappa_n(x,y),$$

introduced by Sloan, is a projector onto $\mathcal{P}_n$. Given any sequence of such $L_n, n \to \infty$, we extend to projective spaces results of Sloan and Reimer on spheres and give simplified proofs of related results of Mhaskar.

Specifically, given a projective space $M$, our main results are

1. Point spacing: $\exists c$ such that $\forall n \quad M = \bigcup_{y \in Y_n} \{ x : \rho(x,y) \leq cn^{-1} \}$.
2. Lower bounds: $\forall x \in M, ||L_n||_{\infty} := \sup ||f||_{\infty} \leq ||L_n(f)||_{\infty} \geq \sup_x \int |\kappa_n(z,\cdot)|d\mu = \int |\kappa_n(x,\cdot)|d\mu$.
3. Upper bounds: $\exists c_1, c_2$ such that $\forall x \in M, ||L_n||_{\infty} \leq c_2 \int |\kappa_n(x,\cdot)|d\mu \sim c_2 n^{(\dim M - 1)/2}$.

Extensions of (1), (2) and the first inequality in (3) to other $M$ are almost certainly true and we are currently checking the details of possible proofs. Some form of the asymptotic growth rate in (3), probably with $\dim M - 1$ replaced by $\dim M - \text{rank}(M)$ when $\dim M > \text{rank}(M)$, is also likely to be true, at least when the action of $G_x = \{ g \in G : gx = x \}$ on the tangent space $T_x(M)$ is irreducible.

Evaluation of the Joint Spectral Radius
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Evaluation of the joint spectral radius (JSR) of a finite set of square matrices is fundamental for determining Hölder regularity of refineable functions. While, in general, even the estimation of the JSR
is NP hard, exact evaluation is possible for a substantial class of problems. In this talk, we present a novel approach to the topic which is based on depth-first-search in set-valued trees.

**A subclass of the parameterized wavelets of length twelve**

David Roach
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In this talk, a subclass of the length twelve parameterized wavelets is given. This subclass is a parameterization of the coefficients of a subset of the trigonometric polynomials, \( m(\omega) \), that satisfy the necessary conditions for orthogonality, that is \( m(0) = 1 \) and \( |m(\omega)|^2 + |m(\omega + \pi)|^2 = 1 \), but is not sufficient to represent all possible trigonometric polynomials satisfying these constraints. This parameterization has three free parameters whereas the general parameterization would have five free parameters. Finally, we graph some example scaling functions from the parameterization and conclude with a numerical experiment with image compression.

**Interpolatory blending net subdivision schemes of Dubuc-Deslauriers type**

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In this talk we deal with interpolatory net subdivision schemes, i.e. iterative procedures which repeatedly refine nets of univariate functions and converge to continuous bivariate functions interpolating the initial net. We extend the family of Dubuc-Deslauriers interpolatory point subdivision schemes and construct an interpolatory blending net subdivision scheme. To this purpose, at each recursion step we use a net refinement operator based on the evaluation of an interpolating Gordon surface. Doing so the limit surface is not only interpolating the initial net of univariate functions, but also all the nets generated by the iterative procedure.

Convergence and smoothness properties of these blending net subdivision schemes are proved in relation to the properties of the cardinal blending function used to define the Gordon surface, and by exploiting the notion of proximity with the tensor-product Dubuc-Deslauriers schemes for points.

We conclude by presenting an example of a family of interpolatory blending net subdivision schemes whose first two members can be used to design \( C^1 \) and \( C^2 \) surfaces from given nets of 3D curves.

**Architectures for compressive sampling of correlated signals**

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We will discuss several ways in which recent results on the recovery of low-rank matrices from partial observations can be applied to the problem of sampling ensembles of correlated signals. We will present several architectures that use simple analog building blocks (vector-matrix multiply, modulators, filters, and ADCs) to implement different types of measurement schemes with ”structured randomness”. These sampling schemes allow us to take advantage of the (a priori unknown) correlation structure of the ensemble by reducing the total number of observations required to reconstruct the collection of signals. We will discuss scenarios that use an ADC for every channel, and those which multiplex the channels onto a single line which is sampled with a single ADC.
Finite-Term Recurrence Relations for Bergman and Szegő Polynomials

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Utilizing an extension of Havin’s lemma we prove that polynomials orthogonal over the unit disk with respect to certain weighted area measures (Bergman polynomials) cannot satisfy a finite-term recurrence relation unless the weight is radial, in which case the polynomials are simply monomials. For polynomials orthogonal over the unit circle (Szegő polynomials) a simple argument is given to show that the existence of a finite-term recurrence implies that the weight must be the reciprocal of the square modulus of a polynomial.

Open Problems and Recent Progress in Kernel-Based Approximation

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As far as time permits, I shall survey recent progress in research on kernels consisting of the topics 1. Kernel Construction 2. Scaling, with subtopics 2a Natural Scales of Functions in Sobolev Spaces, 2b flat limits, 2c multiscale methods. 3. Choice of Bases, possibly including a review of preconditioning techniques 4. State-of-the-art in error bounds, in particular the exponential ones

Exact Evaluation of Non-Polynomial Subdivision Schemes at Rational Parameter Values

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We describe a method for exact evaluation of a limit mesh defined via subdivision on a uniform grid of any size. Other exact evaluation technique either restrict the grids to have subdivision sampling and are, hence, exponentially increasing in size or make assumptions about the underlying surface being piecewise polynomial. As opposed to exact evaluation techniques like Stam’s method, our approach works for both polynomial and non-polynomial schemes. The values for this exact evaluation scheme can be computed via a simple system of linear equation derived from the scaling relations associated with the scheme or, equivalently, as the dominant left eigenvector of an upsampled subdivision matrix associated with the scheme. To illustrate one possible application of this method, we demonstrate how to generate adaptive polygonalizations of a non-polynomial quad-based subdivision surfaces using our exact evaluation method. Our method guarantees a water-tight tessellation no matter how the surface is sampled and is quite fast. We achieve tessellation rates of over 48.4 million triangles/second using a CPU implementation.

Multimodal Computing and Interaction (M2CI)

Hans-Peter Seidel

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The past three decades have brought dramatic changes in the way we live and work. This phenomenon is widely characterized as the advent of the Information Society. Ten years ago, most digital content was textual. Today, it has expanded to include audio, video, and graphical data. The challenge
is now to interface, organize, understand, and search this multimodal information in a robust, efficient and intelligent way, and to create dependable systems that allow natural and intuitive multimodal interaction.

The Cluster of Excellence on Multimodal Computing and Interaction (M2CI), established by the German Research Foundation (DFG) within the framework of the German Excellence Initiative, addresses this challenge. The term multimodal describes the different kinds of information such as text, speech, images, video, and graphics, and the way it is perceived and communicated, particularly through vision, hearing, and human expression.

In this presentation I will briefly elaborate on the structure of this research cluster, and I will then highlight some of our ongoing research by means of examples. Topics covered include 3D Reconstruction and Digital Geometry Processing, Motion and Performance Capture, 3D Video Processing, and Multimodal Music Processing.

Speaker’s Biography: Hans-Peter Seidel is the scientific director and chair of the computer graphics group at the Max Planck Institute (MPI) for Informatics and a professor of computer science at Saarland University, Saarbrucken, Germany. He is co-chair of the Max Planck Center for Visual Computing and Communication (MPC-VCC) (since 2003), and he is the scientific coordinator of the Cluster of Excellence on Multimodal Computing and Interaction (MMCI) that was established by the German Research Foundation (DFG) within the framework of the German Excellence Initiative in 2007. In addition, Seidel is a member of the Governance Board of the newly established Intel Visual Computing Institute (IVCI) (since 2009).

Reduction and Null Space Algorithms for the Subspace Clustering Problem
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This paper presents two algorithms for clustering high-dimensional data points that are drawn from a union of lower dimensional subspaces. The first algorithm is based on binary reduced row echelon form of a data matrix. It can solve the subspace segmentation problem perfectly for noise free data, however, it is not reliable for noisy cases. The second algorithm is based on Null Space representation of data. It is devised for the cases when the subspace dimensions are equal. Such cases occur in applications such as motion segmentation and face recognition. This algorithm is reliable in the presence of noise, and applied to the Hopkins 155 Dataset it generates the best results to date for motion segmentation. The recognition rates for two and three motion video sequences are 99.15% and 98.85%, respectively.

On Markov–Duffin–Schaeffer inequalities with a majorant
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In this talk, we consider the Markov-type problem of estimating the max-norm $\|p^{(k)}\|$ of the $k$-th derivative of an algebraic polynomial $p$ of degree $n$ under restriction

$$|p(x)| \leq \mu(x), \quad x \in [-1,1],$$

where $\mu$ is a non-negative majorant of the form $\mu(x) = \sqrt{R_s(x)}$, with $R_s \in \mathcal{P}_s$. We prove that, for a wide range of such majorants, we have

$$\|p^{(k)}\| \leq \|\omega_{\mu}^{(k)}\|,$$

where $\omega_{\mu}$ is the so-called snake-polynomial for $\mu$, the one which oscillates most between $\pm \mu$ and which is an analogue of the Chebyshev polynomial $T_n$ for $\mu \equiv 1$. Actually, we prove that the same inequality is valid under the weaker Duffin-Schaeffer-type assumption

$$|p(x)| \leq \mu(x), \quad x \in \delta^* = (\tau_i^*)_{i=0}^n.$$
where $\delta^*$ is the discrete set of $n + 1$ oscillation points of $\omega_\mu$.

ideal extensions of ideal complements

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I will discuss a counterexample to a conjecture of Tomas Sauer that every ideal projector on the space of polynomials of degree N can be restricted to an ideal projector on the space of polynomials of degree N-1 and some closely related problems

Proximity Algorithms for Total-Variation Based Image Models

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This talk presents proximity operator frameworks for the study of the total-variation based image models. We view total-variation as the composition of a convex function ($\ell^1$-norm or $\ell^2$-norm) with the first order difference operator. A characterization of the solutions to the total-variation based image model is given in terms of the proximity operators of the $\ell^1$-norm and $\ell^2$-norm which have explicit expressions. The characterization naturally leads to a fixed-point algorithm for computing a solution of the model.

Recent Asymptotic Expansions Related to Legendre Polynomials

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We present several recent asymptotic expansions that are related to Legendre polynomials $P_n(x)$. We derive the asymptotic expansion as $n \to \infty$ of the error in $n$-point Gauss-Legendre quadrature for integrals $\int_{-1}^{1} f(x) dx$, where $f(x)$ may have arbitrary algebraic-logarithmic endpoint singularities. We next present a full asymptotic expansion for the Legendre polynomials $P_n(x)$ as $n \to \infty$ that is valid for $-1 < x < 1$, and use this to derive a full asymptotic expansion for integrals of the form $\int_{c}^{d} f(x) P_n(x) dx$, where $-1 < c < d < 1$ and $f(x)$ is allowed to have arbitrary algebraic singularities at $x = c$ and $x = d$. Following this, we derive asymptotic expansions for the Legendre series coefficients $e_n[f] = (n + 1/2) \int_{c}^{d} f(x) P_n(x) dx$, where $f(x)$ may have arbitrary interior and endpoint singularities. We show that contributions to these expansions come exclusively from (both sides of) the points of singularity, and that they are expressible in terms the asymptotic expansions of $f(x)$ at these points.

Weak Convergence of CD Kernels: A New Approach

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Let $\mu$ be a probability measure supported on some compact set $K$ in the complex plane and form the set of corresponding orthonormal polynomials $\{p_n(z)\}_{n=0}^\infty$. For every natural number $n$, define

$$d\mu_n(z) = \frac{1}{n + 1} \sum_{j=0}^{n} |p_j(z)|^2 d\mu(z)$$
and let $\nu_n$ be the normalized zero counting measure for the polynomial $p_n$. If $\mu$ is supported on a compact subset of the real line or on the unit circle, we provide a new proof of a 2009 theorem of Simon that for any fixed natural number $j$, the $j^{th}$ moment of $\nu_{n+1}$ and $\mu_n$ differ by at most $O(n^{-1})$ as $n \to \infty$.

**Universality for potential theoretic matrix ensembles**

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We consider ensembles of random matrices with weights formed from equilibrium potentials of compact regions in the complex plane. Assuming the boundary is sufficiently smooth then we prove universality of the reproducing kernel for these ensembles. These results follow from the exterior asymptotics of the orthogonal polynomials associated to the weights.

**Multivariate splines and optimal recovery problems**

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We construct a continuous quadratic polynomial spline of several variables that solves an optimal recovery problem for the class of functions defined on a convex polytope, whose second derivatives in any direction are uniformly bounded, and for a periodic analogue of this class. The information consists of the values and gradients of the function at some finite set of nodes.

**Convexity preserving splines on triangulations**

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Convexity is often required in the design of surfaces. Typically, a nonlinear optimization problem arises, where the objective function controls the fairness of the surface and the constraints include convexity conditions. We consider convex polynomial splines defined on triangulations. In general, convexity conditions on polynomial patches are nonlinear. In order to simplify the optimization problem, it is advantageous to have linear conditions. We show how sets of sufficient linear convexity conditions can be generated for polynomials defined on a triangle. The construction allows us to give a geometric interpretation, and we can easily construct sets of linear conditions that are symmetric.

**Frame Potential minimization for clustering short time series**

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For finding clusters in gene expression data we present a geometric interpretation of the Short Time Series Expression Miner (STEM) by Ernst et al. The proposed distance measure is interpreted as the distance between points on the $(d-2)$-dimensional unit sphere $S^{d-2}$. The choice of cluster representatives is closely related to classical problems in optimization theory, e.g. Tammes’ of Thomson’s problem. Moreover, we propose a new functional which has a data-driven component and connects the choice of cluster representatives to the theory of Finite Unit Norm Tight Frames.
The fundamental problem of Gabor analysis is to determine triples $(g, \alpha, \beta)$ consisting of an $L^2$-function and lattice parameters $\alpha, \beta > 0$, such that the set of functions $\mathcal{G}(g, \alpha, \beta) = \{ e^{2\pi i \beta t} g(\alpha k) : k, l \in \mathbb{Z}\}$ constitutes a frame for $L^2(\mathbb{R})$; in other words, we must determine the set $\mathcal{F}(g) = \{ (\alpha, \beta) : \alpha > 0, \beta > 0, \mathcal{G}(g, \alpha, \beta) \text{ is a frame} \}$.

Under mild conditions on $g$, it is known that $\mathcal{F}(g) \subseteq \{ (\alpha, \beta) : \alpha \beta < 1 \}$. Until now, the full set $\mathcal{F}(g)$ is known precisely only for a small set of functions (and their dilations): the Gaussian, the hyperbolic secant, the one-sided and the two-sided exponential function.

We establish a new connection of the characterization of Gabor frames, namely the Ron-Shen matrix analysis, to the Schoenberg-Whitney conditions of totally-positive functions. Our main result is the following:

**Theorem.** Assume that $g$ is a totally positive function of finite type. Then $\mathcal{G}(g, \alpha, \beta)$ is a frame, if and only if $\alpha \beta < 1$.

Among the examples of totally positive functions of finite type are the one-sided and two-sided exponentials, the truncated power functions $g(t) = e^{-t^r} \chi_{\mathbb{R}_+}$, and others.

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**Two nonlinear sampling problems**

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In this talk, I will discuss nonlinear sampling process termed with instantaneous companding and subsequently average sampling, and local identification of innovation positions of signals with finite rate of innovation.

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**On the existence of Optimal Subspace Clustering Models**

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Given a finite set of vectors $A$ in a Hilbert space $H$, and given a family $C$ of closed subspaces of $H$, the *subspace clustering problem* consists in finding a union of subspaces in $C$ that best approximates (nearest to) the data $F$. This problem has applications and connections to many areas of mathematics, computer science and engineering such as Generalized Principle Component Analysis (GPCA), learning theory, compressed sensing, and sampling with finite rate of innovation. In this paper, we characterize families of subspaces $C$ for which such a best approximation exists. In finite dimensions the characterization is in terms of the convex hull of an augmented set $C^+$. In infinite dimensions however, the characterization is in terms of a new but related notion of contact half-spaces. As an application, the existence of best approximations from invariant families $C$ of unitary representations of abelian groups is derived.

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**Sparsity and Rank Minimization for Subspace Clustering**

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We consider the problem of fitting one or more subspaces to a collection of data points drawn from the subspaces and corrupted by noise/outliers. We pose this problem as a rank minimization problem,
where the goal is to decompose the corrupted data matrix as the sum of a clean dictionary plus a
matrix of noise/outliers. By constraining the dictionary to be self-expressive, i.e., its elements can be
written as a linear combination of other elements, we formulate this problem as one of minimizing the
nuclear norm of the matrix of linear combinations. Our first contribution is to show that, with noisy
data, this problem can be solved in closed form from the SVD of the data matrix. Remarkably, this
is true for both one or more subspaces. A key difference with respect to existing methods is that our
framework results in a polynomial thresholding of the singular values with minimal shrinkage. Indeed, a
particular case of our framework in the case of a single subspace leads to classical PCA, which requires
no shrinkage. In the case of multiple subspaces, our framework provides an affinity matrix that can
be used to cluster the data according to the subspaces. A major difference with respect to existing
methods is that, rather than using the corrupted data as a dictionary, we build this affinity from a
clean dictionary. Moreover, both the clean dictionary and the affinity matrix can be computed in
closed form. In the case of data corrupted by outliers, a closed-form solution appears elusive. We thus
use an augmented Lagrangian optimization framework, which requires a combination of our proposed
polynomial thresholding operator with the more traditional shrinkage-thresholding operator.

Pythagorean-hodograph Cycloidal curves
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Polynomial Pythagorean-hodograph (PH) curves play significant role in the theory as well as in
practical applications of polynomial curves. They are characterized by the property that the Euclidean
norm of their hodograph, called the parametric speed, is also a polynomial. As a consequence, they
have rational tangent, rational offsets, polynomial arc-length, etc. In this talk, Pythagorean-hodograph
cycloidal curves as an extension of PH cubics will be introduced. Their properties will be examined and
a constructive geometric characterization will be established. Further, PHC curves will be applied in the
Hermite interpolation and the closed form solutions will be determined. The asymptotic approximation
order analysis will also be presented, which clearly indicates which interpolatory curve solution should
be selected in practice.

Compressive Inference of Signals and Systems
from Measurements with Spatial and/or Temporal Diversity
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Researchers in the field of Compressive Sensing (CS) have established a solid suite of theory and
algorithms for recovering sparse signals from incomplete measurements. Much of the core theory in
CS, however, still relies on the assumption that the compressive measurement matrix is populated with
independent and identically distributed (i.i.d.) random entries.

In this talk we will overview several problems in which spatial and/or temporal measurement con-
straints impose a fundamental structure within a randomized compressive measurement operator. This
structure prevents the application of standard CS concentration and isometry results. Thus, we will
present a collection of new concentration of measure bounds for several structured operators and dis-
cuss the implications of these bounds for several problems involving signal recovery from distributed
sensor network measurements, identification of high-dimensional dynamical systems from streaming
measurements, and sparse topology identification in a network of interconnected dynamical systems.

This research is joint work with Borhan Sanandaji, Tyrone Vincent, Christopher Rozell, Han Lun
Yap, Jae Young Park, and Armin Eftekhar.
Optimal shift-invariant spaces with additional invariance
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In this talk we consider two classes of PSI spaces with additional invariance - translation invariant spaces and $1/n\mathbb{Z}$-invariant spaces. We give a constructive proof of existence of an optimal PSI space in each of the two classes best approximating a finite family of $L^2$ functions in the sense of least squares.

Splines on Triangulations with Hanging Vertices
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Polynomial spline spaces defined on triangulations with hanging vertices are studied. In addition to dimension formulae, explicit basis functions are constructed, and their supports and stability are discussed. The approximation power of the spaces is also treated.

Polyharmonic and Related Kernels on Manifolds:
Interpolation and Approximation
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This talk will focus on kernel interpolation and approximation in a general setting. It turns out that for a wide class of compact, connected $C^\infty$ Riemannian manifolds, including the important cases of spheres and $SO(3)$, the kernels obtained as fundamental solutions of certain partial differential operators generate Lagrange functions that are uniformly bounded and decay away from their center at a fast algebraic rate, and in certain cases, an exponential rate. This fact has important implications for both interpolation and approximation which will be discussed. The class of kernels considered in this talk include the restricted surface splines on spheres as well as surface splines for $SO(3)$, both of which have elementary closed form representations which are computationally implementable. The talk is based on some recent joint work with T. Hangelbroek and F.J. Narcowich along with previous work with the same co-authors together with X. Sun.

RBF approximation estimates for perturbations of
Green’s functions
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It is known that a Green’s function-type condition may be used to derive rates for approximation by radial basis functions (RBFs). This talk will address a method for obtaining rates for approximation by functions that can be convolved with a finite Borel measure to form a Green’s function. Following a description of the method, rates will be found for two classes of RBFs. Specifically, rates will be found for the Sobolev splines, which are Green’s functions, and the perturbation technique will be employed to determine rates for approximation by Wendland functions.
A Kernel Method for Solving Parabolic Differential Equations on Surfaces
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Kernel methods such as those based on radial basis functions (RBFs) are becoming increasingly popular for numerically solving partial differential equations (PDEs) because they are geometrically flexible, algorithmically accessible, and can be highly accurate. There have been many successful applications of this technique to various types of PDEs defined on planar regions in $\mathbb{R}^1$, $\mathbb{R}^2$, and $\mathbb{R}^3$, and more recently to PDEs defined on the surface of a sphere $S^2$. In this talk we describe a kernel method based on RBFs for numerically solving parabolic PDEs defined on more general surfaces, specifically on smooth, closed embedded submanifolds of $\mathbb{R}^d$. For $d = 3$, these types of problems have received growing interest in the computer graphics, chemistry, and biology communities to model such things as texture mappings, nonlinear chemical oscillators in excitable media, pattern formations on animals, and diffusion of chemicals on biological cells or membranes. Our kernel-based method applies to surfaces that can be described parametrically or implicitly (as the level-set of a smooth function) and only requires nodes at “scattered” locations on the surface. Additionally, it does not rely on any surface-based metrics and avoids any intrinsic coordinate systems, and thus does not suffer from any coordinate singularities. We illustrate the accuracy, stability, and flexibility of the method on various surfaces and test problems from the literature, including some non-linear systems of parabolic PDEs.

Monotone Interpolation using non-linear techniques
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Monotonicity-preserving approximation methods are used in numerous applications. These methods are expected to reconstruct a function from a discrete set of data preserving its monotonicity (i.e., the reconstructed function has to be monotone wherever the discrete data are). Classical monotonicity-preserving methods sacrifice interpolation to get monotonicity. In this work we study the construction of a monotonicity-preserving Hermite interpolant that does not fully sacrifice accuracy. To achieve this we use nonlinear procedures to compute the derivatives. Then, we use a filtering process to keep the approximations that will produce monotonicity-preserving interpolants maintaining, at the same time, the accuracy order as high as possible. We perform several numerical experiments to compare the algorithm against some classical methods.

Asymptotics of Padé approximants to a certain class of elliptic type functions
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Let $a_1$, $a_2$, and $a_3$ be three given non-collinear points. There exists a unique connected compact $\Delta$, called Chebotarëv continuum, containing these points that has minimal logarithmic capacity among all continua joining $a_1$, $a_2$, and $a_3$. It consists of three analytic arcs $\Delta_k$, $k \in \{1, 2, 3\}$, that emanate from a common endpoint, say $a_0$, and end at each of the given points $a_k$. We orient each arc $\Delta_k$ from $a_0$ to $a_k$. According to this orientation we distinguish the left (+) and right (−) sides of each $\Delta_k$. Let $h$ be a complex-valued Dini-continuous non-vanishing function given on $\Delta$. We define the Cauchy integral of $h$ as

$$f_h(z) := \frac{1}{\pi i} \int_{\Delta} \frac{h(t)}{t - z} \frac{dt}{w^+(t)}.$$
where integration is taking part according to the orientation of each $\Delta_k$, that is, from $a_0$ to $a_k$, and $w(z) := \sqrt{\prod_{k=0}^3(z - a_k)}$, $w(z)/z^2 \to 1$ as $z \to \infty$. The function $f_h$ is holomorphic outside of $\Delta$ and vanishes at infinity. In this work we present the results on asymptotic behavior of classical Padé approximants, $\pi_n$, to functions $f_n$ as $n \to \infty$, where $\pi_n$ is a rational function of type $(n-1, n)$ that interpolates $f_h$ at infinity with maximal order.

Compressed sensing with partial support information
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Compressed sensing is a powerful "non-adaptive" signal acquisition paradigm. After making the initial assumption that the high-dimensional signals to be acquired are sparse or compressible, one constructs a universal sampling method (i.e., a measurement matrix) that will provide sufficient information to recover exactly or approximately the underlying signal. Typically, the reconstruction algorithm (e.g., 1-norm minimization) is also non-adaptive – i.e., it does not utilize any additional information about the signal to be recovered and can be used to estimate any sufficiently sparse or compressible signal. In various applications, however, there is prior information that can be exploited to improve the quality of the recovery (this is the basis of various approaches that fall under the term "model-based compressed sensing"). In this talk, we will study such a method that improves signal reconstruction from compressed sensing measurements when partial support information is available. We propose to use a certain weighted 1-norm minimization algorithm in this setting. We prove that if at least 50

Subdivision of manifold-valued data: the single tangent plane versus the two tangent plane approach
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Linear wavelet transforms and their associated linear subdivision schemes are very well-studied. Motivated by the burgeoning of different types of manifold-valued data in areas of science and engineering, for example in diffusion tensor imaging and collaborative motion modeling, a novel framework of nonlinear wavelet transform was introduced for multiscale representation of such data. This development, in turn, motivate us to understand how to adapt a given linear subdivision scheme to manifold-valued data.

For this purpose, the bare minimum we need is something called a ‘retraction map’ on the manifold. A retraction allows us to map back and forth a manifold and its tangent bundle.

A subtle point here is that we can choose to use the same tangent plane for both the odd and even rules in a dyadic subdivision scheme; or we can use two different tangent planes. These two approaches happen to lead to wildly different results.

We shall present a new result for (David Donoho’s) single tangent plane approach: we prove that this approach is always $C^3$ or $C^4$ equivalent to the underlying linear scheme. But unless in rather restrictive cases, the equivalence breakdown once we hit 5th order smoothness. (It almost remind us of Galois theory!)

Parameterized Univariate Compactly Supported Wavelets With Dilation Factor $q \geq 2$ and Image Compression
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In this paper, we give the parameterized filters which satisfy the necessary conditions for orthonormality. The parameterization includes all compactly supported univariate scaling functions with dilation factor $q \geq 2$ contained within the interval $[0, \frac{2q-1}{q-1}]$. Finally, we demonstrate the parameterized
wavelets with dilation factor $q = 3$ of length six and biorthogonal Daubechies 9/7 wavelets in an image compression scheme.

**Digital Shearlet Transform on Pseudo-Polar Grids**

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Directional representative systems provide sparse approximation of anisotropic features are highly desired in both theory and application. The shearlet system is a novel system developed by Guo, Kutyniok, Labate, Lim, Weiss, which achieves these properties. In this talk, we first develop a digital shearlet theory which is rationally designed in the sense that it is the digitalization of the existing shearlet theory for continuum data. This shows that shearlet theory indeed provides a unified treatment of both the continuum and digital realms. Our implementation of the digital shearlet transform is based on the utilization of pseudo-polar grids and the pseudo-polar Fourier transform, which provide a natural implementation for digital shearlets on the discrete image domain. However, the pseudo-polar Fourier transform is generally NOT unitary, hence its adjoint transform cannot be used directly for reconstruction. Unitarity can be achieved by careful weighting of the pseudo-polar grid, yet it is generally difficult to obtain such a weight function. We show how efficient weight functions can be designed and obtained on the pseudo-polar grids so that almost unitarity can be achieved. In addition, we shall discuss the software package ShearLab that implements the digital shearlet transform. The ShearLab in addition provides various quantitative measures allowing one to tune parameters and objectively improve the implementation as well as compare different directional transform implementations. Numerically results and examples will be provided to illustrate our digital shearlet transform.