Hughes, Bruce [Hughes, C. Bruce]; Prassidis, Stratos
Control and relaxation over the circle. (English. English summary)
This well-written research monograph obtains geometric analogues of the Bass-Heller-Swan fundamental theorem of algebraic $K$-theory for a ring $R$:

$$K_1(R[t, t^{-1}]) = K_1(R) \oplus K_0(R) \oplus \tilde{\text{Nil}}(R) \oplus \tilde{\text{Nil}}(R).$$

The Whitehead space $W(X)$ of a finite CW-complex $X$ is a space of homotopy equivalences to $X$ from other finite CW-complexes. The main theorem is a homotopy equivalence:

$$W(X \times S^1) \simeq W(X) \times \Omega^{-1}W(X) \times \tilde{N}(X) \times \tilde{N}(X),$$

There is also such a theorem for the controlled Whitehead space $W(X \times S^1 \to S^1)$ of a compact Hilbert cube manifold $X$:

$$W(X \times S^1 \to S^1) \simeq W(X) \times \Omega^{-1}W(X),$$

as well as for pseudoisotopy spaces. The techniques of proof involve ingenious geometric analogues of the algebraic proof of the original Bass-Heller-Swan result, making much use of controlled topology and manifold approximation fibrations.    A. A. Ranicki (4-EDIN-MS)