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Hughes, Bruce; Ranicki, Andrew

Ends of complexes. (English)

Cambridge Tracts in Mathematics. 123. Cambridge: Cambridge University Press. xxv, 353 p. £45.00; $ 64.95 (1996). [ISBN 0-521-57625-3/hbk]

The book under review is a valuable contribution to the literature in an area of mathematics, geometric topology, where many of the breakthroughs of the last 30-40 years have hitherto been documented only in the original papers. This statement is not intended to imply that the book's content is purely of an expository nature. In fact, there are many new results and new angles on old results. However, it is meant to indicate that exposition, also of older results, is an important agenda for the authors. A prospective reader with some background knowledge in the area of non-compact geometric topology can get a good perspective on the book by reading the chapter summaries which occupy pp. xxii–xxv in the book and which we quote below. Readers with less background may want to start with the Introduction (pp. ix–xxi). It sets the stage quite nicely but it is too long to quote here. It can be found on the Internet at http://www.maths.ed.ac.uk/people/aar/books/index.htm. Even if the reviewer does not feel competent to improve on the authors' own summary of the contents of the book, he does want to rise to the challenge issued by the authors when they write (reviewer's emphasis) "Errata (if any) to this book will be posted in the [above mentioned] WWW Home page." At the time of review only one item is seen there (p. 72, l. -14: "g_*" should be "u_*"). The reviewer offers the following tiny selection to add to the list: p. 2, l. 14: "in" should be removed. p. 27, l. 12: "non-zero divisor" ought to be "non zero-divisor". p. 32, ll. 12, 24: The use of linefeeds in TEX is somewhat inconsistent. Further errata found by the reviewer (if many) will be sent directly to the above homepage.

Chapter summaries (quoted from the book): Part One, Topology at infinity, is devoted to the basic theory of the general, geometric and algebraic topology at infinity of non-compact spaces. Various models for the topology at infinity are introduced and compared.

Chapter 1, End spaces, begins with the definition of the end space $e(W)$ of a non-compact space W. The set of path components $\pi_0(e(W))$ is shown to be in one-to-one correspondence with the set of ends of W (in the sense of Definition 1 above) for a wide class of spaces.

Chapter 2, Limits, reviews the basic constructions of homotopy limits and colimits of spaces, and the related inverse, direct and derived limits of groups and chain complexes. The end space $e(W)$ is shown to be weak homotopy equivalent to the homotopy inverse limit of cocompact subspaces of W and the homotopy inverse limit is compared to the ordinary inverse limit. The fundamental group at infinity $\pi_1^\infty(W)$ if W is defined and compared to $\pi_1(e(W))$.

Chapter 3, Homology at infinity, contains an account of locally finite singular homology, which is the homology based on infinite chains. The homology at infinity $H_*^\infty(W)$ of a space W is the difference between ordinary singular homology $H_*(W)$ and locally finite

singular homology $H_*^{lf}(W)$.
Chapter 4, Cellular homology, reviews locally finite cellular homology, although the technical proof of the equivalence with locally finite singular homology is left to Appendix A.
Chapter 5, Homology of covers, concerns ordinary and locally finite singular and cellular homology of the universal cover (and other covers) $\widetilde{W}$ of W. The version of the Whitehead theorem for detecting proper homotopy equivalences of CW complexes is stated.
Chapter 6, Projective class and torsion, recalls the Wall finiteness obstruction and Whitehead torsion. A locally finite finiteness obstruction is introduced, which is related to locally finite homology in the same way that the Wall finiteness obstruction is related to ordinary homology, and the difference between the two obstructions is related to homology at infinity.
Chapter 7, Forward tameness, concerns a tameness property of ends, which is stated in terms of the ability to push neighbourhoods towards infinity. It is proved that for forward tame W the singular chain complex of the end space $e(W)$ is chain equivalent to the singular chain complex at infinity of W, and that the homotopy groups of $e(W)$ are isomorphic to the inverse limit of the homotopy groups of cocompact subspaces of W. There is a related concept of forward collaring.
Chapter 8, Reverse tameness, deals with the other tameness property of ends, which is stated in terms of the ability to pull neighbourhoods in from infinity. It is closely related to finite domination properties of cocompact subspaces of W. There is a related concept of reverse collaring.
Chapter 9, Homotopy at infinity, gives an account of proper homotopy theory at infinity. It is shown that the homotopy type of the end space, the two types of tameness, and other end phenomena are invariant under proper homotopy equivalence at infinity. It is also established that in most cases of interest a space W is forward and reverse tame if and only if W is bounded homotopy equivalent at ∞ to $e(W) \times [0, \infty)$, in which case $e(W)$ is finitely dominated.
Chapter 10, Projective class at infinity, introduces two finiteness obstructions which the two types of tameness allow to be defined. The finiteness obstruction at infinity of a reverse tame space is an obstruction to reverse collaring. Likewise, the locally finite finiteness obstruction at infinity of a forward tame space is an obstruction to forward collaring. For a space W which is both forward and reverse tame, the end space $e(W)$ is finitely dominated and its Wall finiteness obstruction is the difference of the two finiteness obstructions at infinity. It is also proved that for a manifold end forward and reverse tameness are equivalent under certain fundamental group conditions.
Chapter 11, Infinite torsion, contains an account of the infinite simple homotopy theory of Siebenmann for locally finite CW complexes. The infinite Whitehead group of a forward tame CW complex is described algebraically as a relative Whitehead group. The infinite torsion of a proper homotopy equivalence is related to the locally finite finiteness obstruction at infinity. A CW complex W is forward (resp. reverse) tame if and only if $W \times S^1$ is infinite simple homotopy equivalent to a forward (resp. reverse) collared CW complex.
Chapter 12, Forward tameness is a homotopy pushout, deals with Quinn's characterization of forward tameness for a σ-compact metric space W in terms of a homotopy

property, namely that the one-point compactification W^∞ is the homotopy pushout of the projection $e(W) \to W$ and $e(W) \to \{\infty\}$, or equivalently that W^∞ is the homotopy cofibre of $e(W) \to W$.
Part two, Topology over the real line, concerns spaces W with a proper map $d\colon W \to \mathbb{R}$.
Chapter 13, Infinite cyclic covers, proves that a connected infinite cyclic cover $\overline{W}$ of a connected compact ANR W has two ends $\overline{W}^+$, $\overline{W}^-$, and establishes a duality between the two types of tameness: $\overline{W}^+$ is forward tame if and only if $\overline{W}^-$ is reverse tame. A similar duality holds for forward and reverse collared ends.
Chapter 14, The mapping torus, works out the end theory of infinite cyclic covers of mapping tori.
Chapter 15, Geometric ribbons and bands, presents bands and ribbons. It is proved that $(M, c\colon M \to S^1)$ with M a finite CW complex defines a band (i.e. the infinite cyclic cover $\overline{M} = c^*\mathbb{R}$ of M is finitely dominated) if and only if the ends $\overline{M}^+, \overline{M}^-$ are both forward tame, or both reverse tame. The Siebenmann twist glueing construction of a band is formulated for a ribbon $(X, d\colon X \to \mathbb{R})$ and an end-preserving homeomorphism $h\colon X \to X$.
Chapter 16, Approximate fibrations, presents the main geometric tool used in the proof of the uniformization Theorem 19 (every tame manifold end of dimension ≥ 5 has a neighbourhood which is the infinite cyclic cover of a manifold band). It is proved that an open manifold W of dimension ≥ 5 is forward and reverse tame if and only if there exists an open cocompact subspace $X \subseteq W$ which admits a manifold approximate fibration $X \to \mathbb{R}$.
Chapter 17, Geometric wrapping up, uses the twist glueing construction with $h = 1\colon X \to X$ to prove that the total space X of a manifold approximate fibration $d\colon X \to \mathbb{R}$ is the infinite cyclic cover $X = \overline{M}$ of a manifold band (M, c).
Chapter 18, Geometric relaxation, uses the twist glueing construction with $h =$ covering translation: $\overline{M} \to \overline{M}$ to pass from a manifold band (M, c) to an h-cobordant manifold band (M', c') such that $c'\colon M' \to S^1$ is a manifold approximate fibration.
Chapter 19, Homotopy theoretic twist glueing, and Chapter 20, Homotopy theoretic wrapping up and relaxation, extend the geometric constructions for manifolds in Chapters 17 and 18 to CW complex bands and ribbons. Constructions in this generality serve as a bridge to the algebraic theory of Part Three. Moreover, it is shown that any CW ribbon is infinite simple homotopy equivalent to the infinite cyclic cover of a CW band, thereby justifying the concept.
Part Three, The algebraic theory, translates most of the geometric, homotopy theoretic and homological constructions of Parts One and Two into an appropriate algebraic context, thereby obtaining several useful algebraic characterizations.
Chapter 21, The algebraic theory, translates most of the geometric, homotopy theoretic and homological constructions of Parts One and Two into an appropriate algebraic context, thereby obtaining several useful algebraic characterizations.
Chapter 21, Polynomial extensions, gives background information on chain complexes over polynomial extension rings, motivated by the fact that the cellular chain complex of an infinite cyclic cover of a CW complex is defined over a Laurent polynomial extension.
Chapter 22, Algebraic bands, discusses chain complexes over Laurent polynomial extensions which have the algebraic properties of cellular chain complexes of CW complex bands.

Chapter 23, Algebraic tameness, develops the algebraic analogues of forward and reverse tameness for chain complexes over polynomial extensions. This yields an algebraic characterization of forward (and reverse) tameness for an end of an infinite cyclic cover of a finite CW complex. End complexes are also defined in this algebraic setting.
Chapter 24, Relaxation techniques, contains the algebraic analogues of the constructions of Chapters 18 and 20. When combined with the geometry of Chapter 18 this gives an algebraic characterization of manifold bands which admit approximate fibrations to S^1.
Chapter 25, Algebraic ribbons, explores the algebraic analogue of CW ribbons in the context of bounded algebra. The algebra is used to prove that CW ribbons are infinite simple homotopy equivalent to infinite cyclic covers of CW bands.
Chapter 26, Algebraic twist glueing, proves that algebraic ribbons are simple chain equivalent to algebraic bands.
Chapter 27, Wrapping up in algebraic K- and L-theory, describes the effects of the geometric constructions of Part Two on the level of the algebraic K- and L-groups.
Part Four consists of the three appendices: Appendix A, Locally finite homology with local coefficients, contains a technical treatment of ordinary and locally finite singular and cellular homology theories with local coefficients. This establishes the equivalence of locally finite singular and cellular homology for regular covers of CW complexes. Appendix B, A brief history of end spaces, traces the development of end spaces as homotopy theoretic models for the topology at infinity. Appendix C, A brief history of wrapping up, outlines the history of the wrapping up compactification procedure.

H.J.Munkholm (Odense)

Keywords : Complex; wrapping; torsion; tameness; ribbons; bands; twist glueing; ends; fundamental group at infinity; homology at infinity; Whitehead theorem; proper homotopy; Wall finiteness obstruction
Classification :
∗57-02 Research monographs (manifolds)
55-02 Research monographs (algebraic topology)

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