Math 4710/6710 – Graph Theory – Fall 2019

Extra problems (not from the book)
and extra information on problems from the book

X11, modified version of B&M (2nd pr.) 14.7.2(b). (a) Not assigned.

(b) Calculate the chromatic polynomial of the 4-cycle C_4 by using the recursion $P(G\backslash e, x) = P(G, x) + P(G/e, x)$ to express it as an integer linear combination of chromatic polynomials of complete graphs. [The formula here is equivalent to formula (E) from class, so use formula (E) if you prefer.] Expand and simplify your answer, giving a final answer in the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_1 x + a_0$.

X12. Use the deletion-contraction formula for the chromatic polynomial to reduce $P(C_5, x)$ to a combination of chromatic polynomials of trees, and hence to give it in the form $a_n x^n + a_{n-1}x^{n-1} + \ldots + a_0$.

X13, modified version of B&M (2nd pr.) 10.3.2. Let G be a connected planar graph that has a cycle. Let k be the girth (length of the shortest cycle) of G, and assume that $k \geq 3$.

(a) Show that $m \leq k(n-2)/(k-2)$. [You may assume the (nontrivial) fact that the boundary of every face in a planar embedding of G contains a cycle.]

(b) Deduce that $K_{3,3}$ is nonplanar.