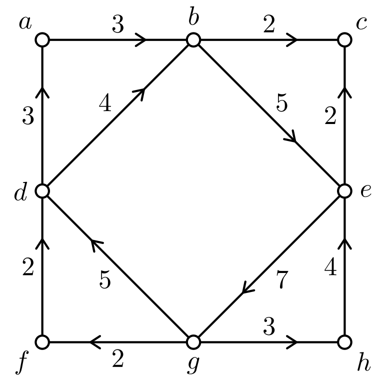
Math 4710/6710 – Graph Theory – Fall 2019

Extra problems (not from the book)
and extra information on problems from the book

X7. Use the algorithm described in class to decompose the flow f shown in the digraph below into a positive linear combination of flows along directed cycles and directed paths. Document your working as follows. At each step show the current remaining flow on a copy of the graph, highlight the edges of the cycle or path along which you are removing flow, say how much flow you are removing along that cycle or path, and then show the new flow on a new copy of the graph. Summarize your results by writing $f = \alpha_1 \chi_{C_1} + \ldots + \alpha_s \chi_{C_s} + \beta_1 \chi_{P_1} + \ldots + \beta_t \chi_{P_t}$ where you give explicit values for $\alpha_1, \ldots, \alpha_s, \beta_1, \ldots, \beta_t$ and explicit descriptions of $C_1, \ldots, C_s, P_1, \ldots, P_t$.

Based on your decomposition, show on two copies of the digraph a nonnegative circulation f_C and a nonnegative acyclic flow f_A that sum to f.



X8. Use the Flow Decomposition Theorem to prove the following for a network $N = (D, c)$.

(a) For every feasible xy-flow f_0, there is a feasible acyclic xy-flow f_A of equal value.

(b) Every nonnegative acyclic xy-flow f_0 of positive value can be written as a positive linear combination of flows along directed xy-paths.

X9. (a) By assigning appropriate flow values and using the Flow Decomposition Theorem, show that every digraph without isolated vertices such that every vertex v has $d^+(v) = d^-(v)$ can be written as the union of arc-disjoint directed cycles.

(b) By orienting the edges appropriately and using (a), show that every nontrivial eulerian (undirected) graph is the union of edge-disjoint cycles.

X10. Suppose G is a graph with an even number of vertices n.

(a) Using the result from class on the parity of shortfall (which is also B&M (2nd pr.) 16.3.2), prove that if $c_{\text{odd}}(G - S) < |S|$ for some $S \subseteq V(G)$, then we actually have $c_{\text{odd}}(G - S) \leq |S| - 2$.

(b) Suppose that $c_{\text{odd}}(G - S) < |S|$ for every $S \subseteq V(G)$ with $|S| \geq 2$. Let xy be an arbitrary edge of G. Prove that G has a perfect matching containing xy. [Hint: use (a) and apply Tutte's Theorem to $G - \{x, y\}$.]