

Math 4710/6710 – Graph Theory – Fall 2019

Extra problems (not from the book)  
and extra information on problems from the book

**X5.** Suppose  $D$  is a connected digraph. Prove that if  $D$  has a circulation  $f$  that is positive ( $f(a) > 0 \forall a \in A(D)$ ) then  $D$  is strongly connected.

**X6.** In the network below, use the Ford-Fulkerson algorithm, starting from the zero flow, to find a maximum flow from  $c$  to  $g$ . You may augment along more than one path at each step if the paths are arc-disjoint. At each step you should **explicitly show the residual network**, show in the residual network the path or paths you are augmenting along, say by how much you are augmenting along each path, and then show the new flow. Prove that your answer is optimal by finding the set reachable from  $c$  in the residual network, and using that to find a corresponding minimum cut.

