2.14. Find the number of integral solutions to

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 12$$

with $x_1, x_2, x_3 \geq 0$, $x_4 \geq 1$, $2 \leq x_5 \leq 7$ and $3 \leq x_6 \leq 9$.

2.15. Find the number of ways to distribute 27 balls into 5 boxes with the following restrictions: box 1 has at most 10 balls and is not empty, box 2 has at least 3 and at most 7 balls, box 3 has at least 3 balls, box 4 has at most 8 balls, and there are no restrictions on box 5.

2.16. Find the number of solutions of $x_1 + x_2 + ... + x_k = n$, where $k \geq 2$, x_1 and x_2 are arbitrary nonnegative integers, and x_3, x_4, ..., x_k are nonnegative <u>even</u> integers.

2.17. Find the number of solutions of $x_1 + x_2 + ... + x_{3k} = n$, where $x_i \equiv i \bmod 3$ for $1 \leq i \leq 3k$, and each x_i is a positive integer.

2.18. Find the number of ways to distribute 35 identical balls into 5 distinct boxes subject to the following restrictions: boxes 1 and 2 each **cannot** have 0, 1, 2, 7, 8, 9 or 10 balls; box 3 has at least 2 and at most 9 balls; and boxes 4 and 5 are unrestricted. Give an exact number. [Hint: express the generating function for each box as something over $1 - x$.]

2.19. Find the number of k-subsets of $\mathbb{N}_n$ so that if the elements are $a_1 < a_2 < ... < a_k$, then $a_i \equiv i \bmod 2$. (In other words, a_i is odd when i is odd, and even when i is even.)

2.20. Find the number of 4-subsets $\{a_1, a_2, a_3, a_4\}$ $(a_1 < a_2 < a_3 < a_4)$ of $\mathbb{N}_{20}$ with $a_i + i \leq a_{i+1} \leq a_i + 2i$, $1 \leq i \leq 3$.

2.21. Find the number of subsets of size k, $k \geq 3$, in $\mathbb{N}_n$ for which the smallest three elements are odd and the rest are even. (Be careful: $k = 3$ is a special case.)

2.22. Find the number of k-subsets, $k \geq 1$, of $\mathbb{N}_n$ in which either all elements are even, or all elements are odd. Do it two ways: by generating functions, and by a simple counting method.