Math 4700/6700 – Combinatorics – Spring 2019

Paper

The term paper for the course is to be on a topic in combinatorics that we will not cover in class. A list of suggested topics is below. You are also welcome to choose a topic of your own to do your paper on, or modify one of the suggested topics (make it narrower or broader). The paper requires a proposal and a final version.

I would like to have a wide range of topics covered by the class. If more than two people propose the same topic, some people may be randomly chosen and asked to change topics.

Graduate credit: Students taking Math 6700 are expected to do a longer and more sophisticated paper than students taking Math 4700, and papers will be graded accordingly.

Proposal: You are to submit a hard copy proposal of about 300 words, with several references (three as an absolute minimum, but five is preferred), describing your topic. The point of the proposal is to show that you have thought about your topic, have found some references, and have a general idea of what is involved. It is also to show that you know how to construct a list of references. Most of your references should be **formally published items**; this is explained more fully below.

The whole proposal, including references, must fit on a **single page**. This is due in class on **Tbursday, 28th February**. The proposal is worth 2% of your course grade. **Before** submitting your proposal you should discuss your topic with me, especially if you are not doing one of the suggested topics.

The proposal is not binding; you may change your topic later in the semester after discussion with me.

Final paper:

- Your final paper should be about 4000 to 5000 words (about 13 to 20 pages if you use an 11 point font and $1\frac{1}{2}$ spacing). It will count for 23% of your course grade. It is due in class on **Thursday, 11th April**.
- Submit **both** an electronic copy and a hard copy of your paper. The paper should be typed. Pages should be numbered. The electronic version should be emailed to me as a PDF file. The hard copy should be submitted to me in person, or in an envelope in my mailbox in the Math Office, SC 1326. Make sure pages are stapled together in the top left corner. Do not provide covers or a folder. Pages may be printed on both sides.

Honor code: You may obtain assistance from others in locating information on your topic. However, all writing must be your own work. Sources must be cited, and material taken verbatim from other sources must be clearly indicated (in quotation marks or blocked off) and cited appropriately.

Content: Your paper should choose an area of combinatorics and give me some or all of the following:

- Basic definitions in the area. (You do not need to define things we have covered in class, just new concepts.)
- Some idea of the history of the area. Who first looked at it and when? Why were they interested in it? What were the first problems posed and results proved?
- The most important results in the area. Who proved them, and why are they significant?
- Proofs of some fundamental or important results. If these are too long, you can instead give a short outline of how the results are proved.
- Examples illustrating the concepts and results.
- If you have room, discussion of important subareas of the area.

Citations: All sources you use in preparing the paper should be given in a list of references in alphabetical order by author at the end of the paper, and appropriate citations should be given in the paper itself by referring to this list. (Don't overdo it: if you use multiple facts from a source in a single paragraph, you can make this clear without citing the source multiple times.) Do not put citation information in footnotes.

You should give me the *original* citation for any important results. Do not just cite a textbook; find out who first proved it and give me the citation for the original paper. (But you should still cite textbooks for things like definitions and any other things you take directly from them, such as a proof.)

Most of your citations should be for formally published items. This means books, journal articles (electronic journals are fine), conference proceedings, and so on. You may have a small number of citations

that are web pages (including Wikipedia pages) or other informal items. Note that formally published items may be online or not; the key issue is whether they are published by some source that exercises some kind of quality control.

You are not required to use any particular citation style but your references should contain full information that would allow a reader to locate the item. If you are not sure what this includes, look at the references in books or journal articles, look at the information *MathSciNet* (see below) provides, or look at one of the standard citation styles. I want you to give a full URL (web address) for any non-formally published item you obtained online, even though some standard citation styles do not require this.

Information resources:

1. The Science & Engineering Library has a number of general books on combinatorics. Looking at chapters in these may give you more ideas for project topics. To find these books, and also others on more specific topics, you should go down to the library and browse on the shelves between QA164 and QA166.

2. You are encouraged to use online resources to get started. However, remember that things like Wikipedia, class notes you find online, etc. are not considered authoritative references. As stated above, almost all of your final references should be formally published material.

3. You may want to use the *MathSciNet* database, which is available from the library's web pages, to track down references; note that the classification code for combinatorics is `05`.

4. Google Scholar may be another useful way to find papers and also to track down other papers that cite the ones you have already found. If you go into Google Scholar from the 'Databases' tab of the 'Search Library' link on the library's web site and log in with your VUNet ID, you can access journal papers directly inside Google Scholar.

5. You can also ask the reference librarians in the Science and Engineering Library for help in finding information. In particular, they can help you use MathSciNet. They may also suggest sources of information that had not occurred to you.

Suggested topics:

You are also welcome to come up with your own topic. If you do, please discuss your topic with me.

- Bell-ringing and permutations (change-ringing)
- Counting labelled trees (matrix-tree theorem, Cayley's formula)
- Algorithms for generating permutations, subsets, etc.
- Additive combinatorics: connections between combinatorics and number theory
- Partitions of integers, the pentagonal number theorem, etc.
- Inclusion/exclusion and graph reconstruction
- Permutation statistics: counting permutations with given patterns
- Catalan numbers and the many things they count
- Dirichlet generating functions and number theory
- Sperner's Lemma (on colourings of triangulations) and applications
- Strongly regular graphs, distance-regular graphs and association schemes
- Weight enumerators for error-correcting codes
- Analytic combinatorics (asymptotic expansions)
- The Erdős-Ko-Rado theorem and extremal set theory
- Wilf and Zeilberger's 'Snake Oil' and 'WZ' methods for proving identities
- Young tableaux
- Latin squares
- Balanced incomplete block designs
- Combinatorial games (nim, etc.)
- Asymptotic properties of random graphs
- The history of Polya-Redfield enumeration
- Finite geometries
- The gamma function and related functions
- Steiner triple systems
- Enumeration of maps
- Random walks in graphs
- Ramsey theory
- Room squares
- Hadamard matrices
- The combinatorial Nullstellensatz
- Symmetric functions
- Generating functions in probability
- Tutte polynomials