1. [5] Consider the plane \( \pi \) and line \( l \) as follows;
\[
\pi: 2x - 3y + 4z = 28; \quad l: x = 2t - 1, \quad y = t + 3, \quad z = 15 - t \quad \text{for} \quad t \in \mathbb{R}.
\]
\[
\hat{n} = \left< -1, 3, 15 \right>, \quad \hat{t} = \left< 2, 1, -1 \right>.
\]
(a) Is \( l \) parallel to \( \pi \)? Show your reasoning.

Normal to \( \pi \), \( \hat{n} = \left< 2, -3, 4 \right> \). Parallel to \( l \), \( \hat{t} = \left< 2, 1, -1 \right> \).

If \( l \) is parallel to \( \pi \) then \( \hat{t} \perp \hat{n} \), i.e. \( \hat{t} \cdot \hat{n} = 0 \). Is it?

\[
\hat{t} \cdot \hat{n} = 4 - 3 - 4 = -3 \neq 0 \quad \text{so} \quad l \quad \text{is not parallel to} \quad \pi.
\]

(b) Is \( l \) contained in \( \pi \), disjoint from \( \pi \), or does \( l \) intersect \( \pi \) at a single point? If \( l \) intersects \( \pi \) at a single point, find the point.

Substitute points of \( l \) into eqn for \( \pi \):
\[
\begin{align*}
2(2t-1) - 3(t+3) + 4(15-t) = 28 \\
4t - 2 - 3t - 9 + 60 - 4t = 28 \\
-3t = 28 + 2 + 9 - 60 = -21
\end{align*}
\]
\[
t = 7 \quad \text{is one value of} \quad t, \quad \text{so intersects at single point: putting} \ t = 7 \ \text{get} \ (x, y, z) = \left( 13, 10, -2 \right).
\]

2. [5] Convert \((x, y, z) = (-3, -3, -3\sqrt{6})\), showing your reasoning, into:

(a) cylindrical coordinates \((r, \theta, z)\):

\[
r = \sqrt{x^2 + y^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2},
\]
\[
\tan \theta = \frac{y}{x} = \frac{-3}{-3} = 1 \quad \Rightarrow \quad \theta = \frac{\pi}{4} \quad \Rightarrow \quad \left( r, \theta, z \right) = \left( 3\sqrt{2}, \frac{\pi}{4}, -3\sqrt{6} \right).
\]

Since \( x < 0, y < 0 \), third quadrant, \( \theta = \frac{3\pi}{4} \)

(b) spherical coordinates \((\rho, \theta, \phi)\):

\[
\rho = \sqrt{x^2 + y^2 + z^2} = \sqrt{9 + 9 + 9 \times 6} = \sqrt{18 + 54} = \sqrt{72} = 6\sqrt{2}
\]
\[
\theta \ (\text{from above}) = \frac{3\pi}{4}
\]
\[
\cos \phi = \frac{z}{\rho} = \frac{-3\sqrt{6}}{6\sqrt{2}} = \frac{-3\sqrt{3}}{2\sqrt{2}} = -\frac{3}{2} \quad \Rightarrow \quad \phi = \cos^{-1} \left( \frac{-3}{2} \right) = \frac{3\pi}{6}
\]
\[
\Rightarrow \ (\rho, \theta, \phi) = \left( 6\sqrt{2}, \frac{3\pi}{4}, \frac{3\pi}{6} \right).
\]

Please turn over...
3. Identify each of the following ten surfaces in $\mathbb{R}^3$ using one of the following one- or two-character codes:

<table>
<thead>
<tr>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>EC</td>
<td>elliptic cylinder</td>
</tr>
<tr>
<td>HC</td>
<td>hyperbolic cylinder</td>
</tr>
<tr>
<td>PC</td>
<td>parabolic cylinder</td>
</tr>
<tr>
<td>E</td>
<td>ellipsoid</td>
</tr>
<tr>
<td>C</td>
<td>cone</td>
</tr>
<tr>
<td>H1</td>
<td>hyperboloid of 1 sheet</td>
</tr>
<tr>
<td>EP</td>
<td>elliptic paraboloid</td>
</tr>
<tr>
<td>H2</td>
<td>hyperboloid of 2 sheets</td>
</tr>
<tr>
<td>HP</td>
<td>hyperbolic paraboloid</td>
</tr>
</tbody>
</table>

Working is not required.

\[ s^2 - y^2 - z^2 = 2 \quad \text{H2} \]
\[ y + 3z^2 = 0 \quad \text{PC} \]
\[ y = -3z^2 \quad \text{parabola in } yz \text{-plane} \]
\[ \text{any } x \text{ OK } : \text{ cyl.} \]

\[ x^2 + y^2 + z^2 = 0 \quad \text{EP} \]
\[ -x^2 + 4y^2 - 3z = 0 \quad \text{HP} \]
\[ \text{no elliptic cross-sections} \]
\[ x = 0: \quad 4y^2 - 3z = 0, \quad 2 = \frac{4}{3}y^2 \text{ par. } \lambda \]
\[ y = 0: \quad -x^2 - 3z = 0, \quad 2 = \frac{1}{3}x^2 \text{ par. } \lambda \]

\[ x^2 + 4z^2 = 9 \quad \text{EC} \]
\[ x^2 + 4z^2 = 9 \quad \text{ellipses in } xz \text{-plane} \]
\[ \text{any } y \text{ OK } : \text{ cyl.} \]
\[ x^2 + y^2 + z^2 = 25 \quad \text{E} \quad (\text{actually, sphere!}) \]
\[ x^2 + y^2 = 25 - z^2 \]
\[ \text{ellipses (circular!) for } |z| \leq 5 \]

\[ x^2 + y^2 - 4z^2 = 10 \quad \text{H1} \]
\[ x^2 + y^2 = 4z^2 + (0) \quad \text{ellipses for all } z \]
\[ \text{(circular!)} \]
\[ -x^2 + 4y^2 + 9z^2 = 0 \quad \text{C} \]
\[ 4y^2 + 9z^2 = x^2 \]
\[ \text{ellipses for all } x, \text{ point at } x = 0 \]

\[ 4x^2 - y^2 = 1 \quad \text{HC} \]
\[ 4x^2 - y^2 = 1 \quad \text{hyp. in } xy \text{-plane} \]
\[ \text{any } z \text{ OK } : \text{ cyl.} \]
\[ 9x^2 + z^2 + 16 = y^2 \quad \text{H2} \]
\[ 9x^2 + 2y^2 = 9 \quad \text{ellipses for } |y| \leq 9 \]