Math 150b-09 Spring 2004 – Test 1 Name: ______________________

- Write answers in spaces provided. The backs of pages may be used for rough work.
- Marks are shown in brackets [].
- Please sign the Honor System pledge on the last page.
- **Show all working and explain all reasoning unless otherwise directed.**
- **Express all answers exactly and as simply as possible unless otherwise directed.**
- **Calculators may not be used. Use of calculators is a violation of the Honor Code.**

60

1.

11:41

1. [10] Evaluate the following indefinite integrals.

[2] (a) $\int (3x^3 + 4x^4 + 5x^5)\,dx$ $= \frac{3x^4}{4} + \frac{4x^5}{5} + \frac{5x^6}{6} + C$

[4] (b) $\int (x+2)(x^2+4x+7)^{7/2}\,dx$

Let $u = x^2 + 4x + 7$

$du = (2x+4)\,dx = 2(x+2)\,dx$

$\frac{1}{2}du = (x+2)\,dx$

$= \int \frac{1}{2} u^{7/2}\,du$

$= \frac{1}{2} \frac{u^{9/2}}{(9/2)} + C$

$= \frac{1}{9} u^{9/2} + C = \frac{1}{9}(x^2+4x+7)^{9/2} + C$

[4] (c) $\int \sin(4\theta + \frac{\pi}{2})\,d\theta$

Let $u = 4\theta + \frac{\pi}{2}$

$du = 4\,d\theta$

$\frac{1}{4}du = d\theta$

$= \int \frac{1}{4} \sin u\,du$

$= -\frac{1}{4}\cos u + C$

$= -\frac{1}{4}\cos(4\theta + \frac{\pi}{2}) + C.$

11:43

2. [16] Evaluate the following definite integrals.

2.

[4] (a) $\int_1^4 (15v^2 + \sqrt{v})\, dv$ $= \int_1^4 (15v^2 + v^{1/2})\, dv$

$$= \left[15\frac{v^3}{3} + \frac{v^{3/2}}{3/2}\right]_{v=1}^{4} = \left[5v^3 + \frac{2}{3}v^{3/2}\right]_{v=1}^{4}$$

$$= \left[5\times 64 + \tfrac{2}{3}\times 8\right] - \left[5\times 1 + \tfrac{2}{3}\times 1\right] = 320 + \tfrac{16}{3} - 5 - \tfrac{2}{3}$$

$$= 315 + \tfrac{14}{3} = 319\tfrac{2}{3} \text{ or } \tfrac{959}{3}$$

[5] (b) $\int_0^{\pi/6} \sec^4 x \tan x\, dx$

Let $u = \sec x$

$du = \sec x \tan x\, dx$

$x = \frac{\pi}{6} \Rightarrow u = \sec\frac{\pi}{6} = \frac{2}{\sqrt{3}}$

$x = 0 \Rightarrow u = \sec 0 = 1$

$$= \int_{x=0}^{\pi/6} \sec^3 x \cdot \sec x \tan x\, dx$$

$$= \int_{u=1}^{2/\sqrt{3}} u^3 \cdot du$$

$$= \left.\frac{u^4}{4}\right|_{u=1}^{2/\sqrt{3}} = \frac{1}{4}\left(\frac{2^4}{\sqrt{3}^4} - 1^4\right) = \frac{1}{4}\left(\frac{16}{9} - 1\right)$$

$$= \frac{1}{4}\left(\frac{7}{9}\right) = \frac{7}{36}$$

(Alt. soln:

Let $u = \tan x$

$du = \sec^2 x\, dx$

$x = \frac{\pi}{6} \Rightarrow u = \tan\frac{\pi}{6} = \frac{\sqrt{3}}{3}$

$x = 0 \Rightarrow u = \tan 0 = 0$

$$= \int_0^{\pi/6} \sec^2 x \tan x \sec^2 x\, dx$$

11:46

$$= \int_0^{\pi/6} (\tan^2 x + 1)\tan x \sec^2 x\, dx$$

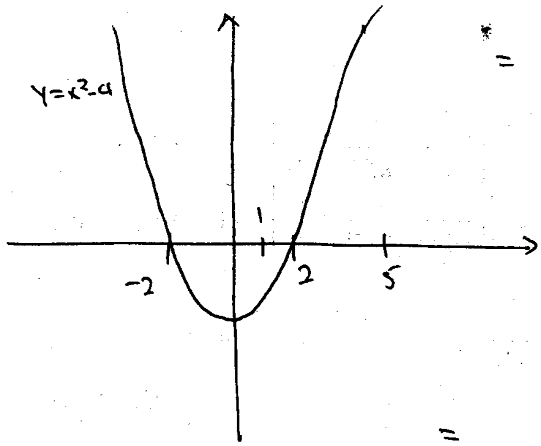
$$= \int_{u=0}^{\sqrt{3}/3} (u^2+1)u\, du = \int_0^{\sqrt{3}/3} (u^3+u)\, du = \left[\frac{u^4}{4} + \frac{u^2}{2}\right]_{u=0}^{\sqrt{3}/3} = \left(\frac{1/9}{4} + \frac{1/3}{2}\right) - (0+0)$$

$$= \frac{1}{36} + \frac{1}{6} = \frac{1+6}{36} = \frac{7}{36}$$

(Also can substitute $u = \cos x$.))

12:13 [7] 2(ctd). (c) $\int_1^5 |x^2-4|\,dx$

$$= \int_1^2 |x^2-4|\,dx + \int_2^5 |x^2-4|\,dx$$



$$\overset{*}{=} \int_1^2 -(x^2-4)\,dx + \int_2^5 (x^2-4)\,dx$$

since $x^2-4\le 0$ for $1\le x\le 2$ Since $x^2-4\ge 0$ for $2\le x\le 5$

$$= \int_1^2 (4-x^2)\,dx + \int_2^5 (x^2-4)\,dx$$

$$= 4x-\tfrac{1}{3}x^3\Big|_{x=1}^{2} + \tfrac{1}{3}x^3-4x\Big|_{x=2}^{5}$$

$$= \left(4(2)-\tfrac{8}{3}\right)-\left(4(1)-\tfrac{1}{3}\right) + \left(\tfrac{125}{3}-4(5)\right)-\left(\tfrac{8}{3}-4(2)\right)$$

$$= 8-\tfrac{8}{3}-4+\tfrac{1}{3}+\tfrac{125}{3}-20-\tfrac{8}{3}+8$$

$$= 8-4-20+8+\left(\frac{-8+1+125-8}{3}\right) = -8+\frac{110}{3}$$

$$= \frac{110-24}{3} = \frac{86}{3} = \underline{28\tfrac{2}{3}}.$$

3. [3] If a particle moves with acceleration $a(t)=8+3\sin t$, where t denotes time, and its velocity at time 0 is $v(0)=7$, find its velocity $v(t)$ at an arbitrary time t.

$a(t) = v'(t) = 8+3\sin t$

$\therefore$ antidifferentiating,

$v(t) = 8t-3\cos t + C$

To find C, use $v(0)=7$:

$7 = v(0) = 8(0)-3\cos(0)+C = -3+C$

$\therefore$ $C=10$

So $v(t) = \underline{8t-3\cos t+10}$.

12:17

[1:51] 3. [5] Evaluate the following derivatives.

[1] (a) $\frac{d}{dx}\int_3^x \sin(w^3+4)\,dw \quad = \quad \sin(x^3+4)$

(straight forward use of FTC 1)

4.

[4] (b) $\frac{d}{dx}\int_5^{x^2+2} \sin^4 t\,dt$: Let $g(x) = \int_5^x \sin^4 t\,dt$, then

$g'(x) = \sin^4 x$ by FTC 1. We want

$$\frac{d}{dx} g(x^2+2) = g'(x^2+2)\cdot \frac{d}{dx}(x^2+2)$$
$$= 2x\, g'(x^2+2)$$
$$= 2x \sin^4(x^2+2).$$

Alt. soln: Let $u = x^2+2$, then

$$\frac{d}{dx}\int_5^{x^2+2} \sin^4 t\,dt = \frac{d}{dx}\int_5^u \sin^4 t\,dt \overset{\text{chain rule}}{=} \frac{du}{dx}\frac{d}{du}\int_5^u \sin^4 t\,dt \overset{\text{FTC1}}{=} 2x.\sin^4 u = 2x\sin^4(x^2+2).$$

4. [6] A child grows at a rate of $\frac{6}{5} + \frac{1}{100}t - \frac{1}{10{,}000}t^2$ cm/month, where t is the child's age in months. How much does the child increase in height between the ages of $2\frac{1}{2}$ years and 5 years? (Be careful!)

$2\frac{1}{2}$ yrs to 5 yrs is $t = \underline{30}$ months to $\underline{60}$ months!

$$\Delta h = \int_{30}^{60} h'(t)\,dt = \int_{30}^{60}\left(\frac{6}{5} + \frac{1}{100}t - \frac{1}{10\,000}t^2\right)dt$$

$$= \frac{6}{5}t + \frac{1}{200}t^2 - \frac{1}{30{,}000}t^3 \Big|_{t=30}^{60}$$

$$= \left(\frac{6\times 60}{5} + \frac{3600}{200} - \frac{216{,}000}{30{,}000}\right) - \left(\frac{6\times 30}{5} + \frac{900}{200} - \frac{27{,}000}{30{,}000}\right)$$

$$= \frac{6\times(\cancel{60-30})}{\cancel{5}} + \frac{36\cancel{00}-9\cancel{00}}{2\cancel{00}} + \frac{27{,}\cancel{000} - 216{,}\cancel{000}}{30{,}\cancel{000}}$$

$$= 36 + \frac{27}{2} + -\frac{\cancel{189}\ 63}{\cancel{30}\ 10}$$

$$= 36 + 13\tfrac{1}{2} - 6\tfrac{3}{10} = 49\tfrac{1}{2} - 6\tfrac{3}{10} = 43\tfrac{1}{5}\ \underline{\text{cm}}.$$

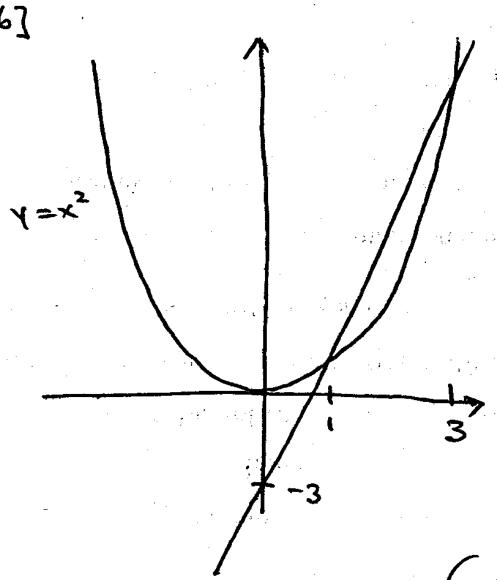
(or $\frac{216}{5}$ cm or 43.2 cm)

11:55

5. [12] (a) Find the area between the curve $y = x^2$ and the line $y = 4x - 3$.

5.

[6]



Curves cross:

$y = x^2 = 4x - 3$

$x^2 - 4x + 3 = 0$

$(x-1)(x-3) = 0$

$x = 1$ or 3

$$\therefore A = \int_1^3 \left(\underbrace{(4x-3)}_{\text{top}} - \underbrace{x^2}_{\text{bottom}} \right) dx$$

$$= \left(2x^2 - 3x - \frac{x^3}{3}\right)\Big|_{x=1}^{3}$$

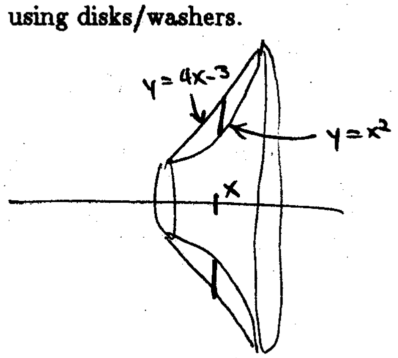
$$= \left(2(3^2) - 3(3) - \frac{27}{3}\right) - \left(2 - 3 - \frac{1}{3}\right)$$

$$= 18 - 9 - 9 - 2 + 3 + \frac{1}{3} = \underline{1\tfrac{1}{3}}.$$

(b) Suppose the area in (a) is rotated around the x-axis to produce a solid of revolution. Set up but **DO NOT EVALUATE** integrals to calculate the volume of this solid ...

(i) using disks/washers.

[3]



Outer radius $R = 4x - 3$

Inner radius $r = x^2$

$A(x) = \pi(R^2 - r^2)$

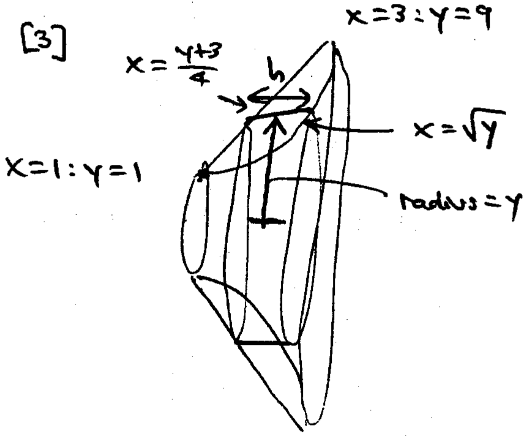
$$V = \int_{x=1}^{3} \pi(R^2 - r^2)\, dx$$

$$= \int_{x=1}^{3} \pi\left((4x-3)^2 - (x^2)^2\right) dx$$

$$= \pi \int_1^3 (16x^2 - 24x + 9 - x^4)\, dx.$$

(ii) using cylindrical shells.

[3]



$$V = 2\pi \int_{y=1}^{9} r(y)\, h(y)\, dy$$

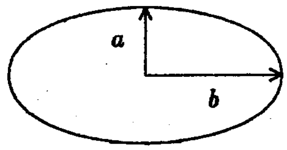
$$= 2\pi \int_1^9 y \cdot \left[\sqrt{y} - \frac{y+3}{4}\right] dy$$

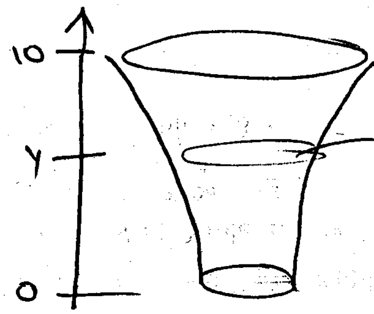
$$= 2\pi \int_1^9 \left[y^{3/2} - \frac{y^2}{4} - \frac{3y}{4}\right] dy.$$

12:01

6. [8] A vase 10 inches high has horizontal cross-sections that are elliptical. The cross-section at height y is an ellipse that has semi-minor axis of length $a = \sec(\frac{\pi}{40}y)$ inches, and a semi-major axis of length $b = 2a$. Given that the area of an ellipse with semi-minor axis a and semi-major axis b is πab, find the volume of the vase.

6.





$$A(y) = \pi a b = \pi a(2a) = 2\pi a^2 = 2\pi \sec^2\left(\frac{\pi y}{40}\right)$$

$$V = \int_{y=0}^{10} A(y)\,dy$$

$$= \int_{y=0}^{10} 2\pi \sec^2\left(\frac{\pi y}{40}\right) dy$$

Let $u = \frac{\pi y}{40}$

$du = \frac{\pi}{40} dy$

$\frac{40}{\pi} du = dy$

$y = 10 \Rightarrow u = \pi/4$

$y = 0 \Rightarrow u = 0$

$$= \int_{u=0}^{\pi/4} 2\cancel{\pi} \cdot \frac{40}{\cancel{\pi}} \sec^2 u\, du$$

$$= 80 \int_0^{\pi/4} \sec^2 u\, du$$

$$= 80 \tan u \Big|_{u=0}^{\pi/4} = 80\left(\tan\frac{\pi}{4} - \tan 0\right) = 80(1-0) = 80 \text{ in}^3.$$

12:03

I pledge on my honor that I have neither given nor received improper aid on this test or quiz.

Signed:

21 min