

Math 3890: Dynamical Systems – Assignment 11

Due in-class on Wednesday, April 17

This assignment has 4 questions for a total of 40 points.

1. (10 points) Compute the topological entropy of the “quadratic map” $f: [0, 1] \rightarrow [0, 1]$ given by $f(x) = 4x(1 - x)$.
2. (10 points) Compute the topological entropy of the map $f: \mathbb{T}^3 \rightarrow \mathbb{T}^3$ given by

$$f(x, y, z) = (x, x + y, y + z) \pmod{1}.$$

(Here $\mathbb{T}^3 = \mathbb{R}^3/\mathbb{Z}^3$ is defined analogously to $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$ or the circle \mathbb{R}/\mathbb{Z}).

3. (10 points) Give an example of continuous maps $f, g: X \rightarrow X$ of a compact space X for which $h(f \circ g) < h(f) + h(g)$. (In contrast: last assignment we saw that $h(f \circ f) = h(f) + h(f)$).
4. (10 points) Let $f: \mathbb{T}^2 \rightarrow \mathbb{T}^2$ be a hyperbolic toral automorphism associated to the matrix $A \in \text{SL}(2, \mathbb{Z})$. [Recall that “hyperbolic” means the eigenvalues of A do not have absolute value 1.] Describe the periodic points of f .