Math 3890: Dynamical Systems – Assignment 1
Due in-class on Wednesday, January 16

This assignment has 5 questions for a total of 58 points.

1. For $n \in \mathbb{N}$, a_n denote the the *last three digits digit* of the number 2^n (that is, a_n is the remainder when 2^n is divided by 1000).
   (a) (7 points) Prove that the sequence $\{a_n\}$ (starting with $n = 3$) is periodic with period at most 100.
   (b) (7 points) Prove that $a_n + a_{n+50} = 1000$ for every $n \geq 3$.
2. Fix $p, q \in \mathbb{N}$ relatively prime and let $b_n \in \{0, \ldots, 9\}$ denote the *ones digit* of the number $\frac{p}{q}n^2$.
   (a) (7 points) Prove that the sequence $\{b_n\}$ is periodic with period at most $10q$.
   (b) (7 points) Prove that the initial $10q + 1$ terms $b_0, b_1, \ldots, b_{10q}$ form a palindromic string.
3. (10 points) Let let d_1 and d_2 be equivalent metrics on a set Y. Prove that a subset $U \subset Y$ is open with respect to the metric d_1 if and only if it is open with respect to the metric d_2

Below, let (X, d) be a metric space. Recall that for a subset $Y \subset X$, the *closure* of Y is the intersection $\overline{Y}$ of all closed subsets of X that contain Y, and the *interior* of Y is the union $\mathring{Y}$ of all open subsets of Y that are contained in Y. Recall also that Y' denotes the set of *accumulation points* (aka limit points) of Y, and that $Y^c = X \setminus Y$ denotes the *complement* of Y.

4. (10 points) Let A be a subset of X. Prove that $\overline{A} = A \cup A'$
5. (10 points) Let A be a subset of X. Prove that $\mathring{A} = \overline{A^c}^c$