Math 3200: Intro to Topology – Homework 13
Due (at the start of class): Thursday, April 21

This assignment has 9 questions for a total of 100 points. Justify all your answers.

Problems in this assignment refer to the following small table of knots:

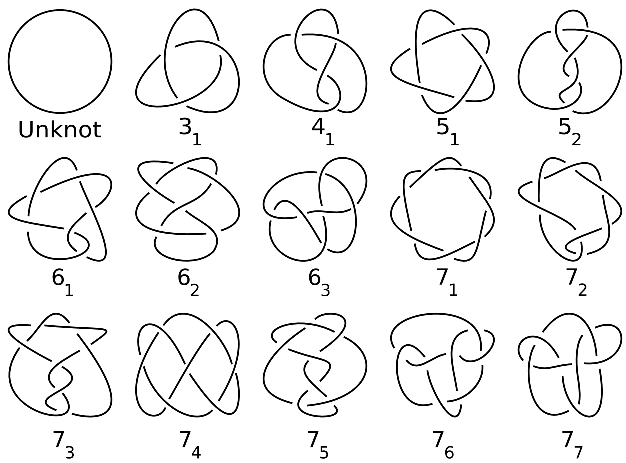
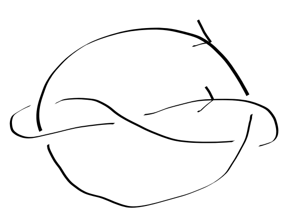


Figure 1: Knot table; licensed under Public Domain via Wikimedia Commons – `http://commons.wikimedia.org/wiki/File:Knot_table.svg#mediaviewer/File:Knot_table.svg`

1. (10 pts) We have seen that knots 3_1 and 6_1 are 3-colorable. Fine one more knot in the table above that is 3-colorable and show your coloring.
2. (10 pts) Prove the negative curl elimination formula for the bracket polynomial. That is, if D is a diagram that contains a negative curl and E is the diagram obtained from D by eliminating that curl, then $\langle D\rangle = -A^{-3}\langle E\rangle$.
3. (10 pts) Show that if the link diagram L is changed to L' by elimination of a negative curl, then the Kauffman polynomials of L and L' are equal; that is $X_L = X_{L'}$.
4. (10 pts) Calculate the bracket polynomial of the Figure Eight knot 4_1.
5. (10 pts) Calculate the bracket polynomial of the knot 5_1.
6. (10 pts) Calculate the Kauffman polynomial of the knot 5_1.
7. Consider the oriented Whitehead link W:



   (a) (10 pts) Calculate the bracket polynomial $\langle W\rangle$ for the Whitehead link W shown above (you may use our previous calculations for diagrams with fewer crossings).

 assignment continues on next page ...

(b) (10 pts) Calculate the writhe $\omega(W)$ and the Kauffman polynomial X_W for the oriented Whitehead link W shown above. Use your answer, if possible, to decide: Is W oriented isotopic to its mirror image W^*? (Recall that W^* is the oriented diagram obtained by interchanging the overpass and underpass at each crossing of W).

The original **Jones polynomial** V_L of a knot or link L is obtained from the Kauffman polynomial X_L by the substitution $A = t^{-1/4}$.

8. (10 pts) Find the Jones polynomial of the Figure Eight knot 4_1.

9. (10 pts) Use the skein relation to show that the Jones polynomial satisfies the relation

$$t^{-1}V_{L+} - tV_{L-} + (t^{-1/2} - t^{1/2})V_{L_0} = 0,$$

where L_+, L_-, and L_0 are three projections that are identical except at one place where they differ as in the figure below: