Math 3200: Intro to Topology – Homework 12

Due (at the start of class): Thursday, April 14

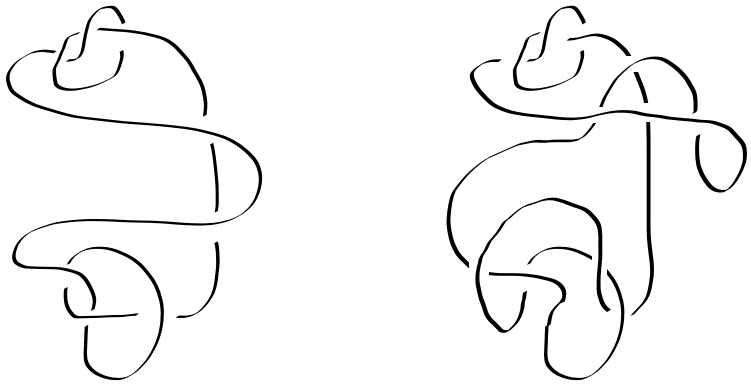
This assignment has 7 questions for a total of 70 points. Justify all your answers.

Recall that an **embedding** is a continuous injection that is a homeomorphism onto its image.

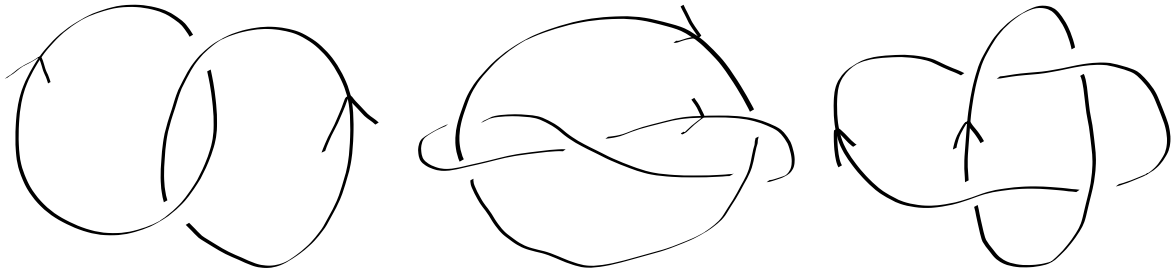
1. (10 pts) Let X and Y be topological spaces. Fix $x_0 \in X$ and let $f\colon Y \to X \times Y$ be the function defined by $f(y) = (x_0, y)$ for $y \in Y$. Prove that f is an embedding.

2. (10 pts) Let I denote the unit interval $I = [0, 1]$ and consider the two embeddings $f, g\colon I \to \mathbb{R}^2$ defined by $f(x) = (x, x)$ and $g(x) = (x^2, x)$. Find an ambient isotopy between f and g.

3. (10 pts) Give a sequence of Reidemeister move from the first diagram to the second. Label each move; R0 moves need not be labeled. PLEASE BE NEAT.



4. (10 pts) Calculate the linking numbers for each of the following (2-component) oriented link diagrams.



a) Hopf link b) Whitehead link c) King Solomon's link

5. (10 pts) Let D_1 be an oriented link diagram with two components C_1 and C_2. Let D_2 be the oriented link diagram with components $\overline{C_1}$ and C_2, where $\overline{C_1}$ is C_1 with the opposite orientation. Prove $lk(D_1) = -lk(D_2)$.

6. (10 pts) Prove that the *absolute value* of the linking number of an oriented link with two components does not depend on the orientation. That is, no matter how the components are oriented, the absolute value of the linking number will be the same.

assignment continues on next page ...

7. (10 pts) Use Problem 6 to determine whether the absolute of the linking number can be used to determine if the following *un*oriented links are isotopic or not:
   (a) Hopf link and Whitehead link
   (b) Hopf link and King Solomon's link
   (c) Whitehead link and King Solomon's link
   (d) Whitehead link and the unlink with two components U_2
   (e) King Solomon's link and U_2.