Math 3200: Intro to Topology – Homework 3
Due (at the start of class): Thursday, January 28

This assignment has 9 questions for a total of 90 points.

1. (10 points) Give an example of an infinite collection of open subsets of $\mathbb{R}^m$ (with the standard metric) whose intersection is not open.

2. (10 points) Give an example of an infinite collection of closed subsets of $\mathbb{R}^m$ (with the standard metric) whose union is not closed.

In questions 3–7, let (X, d) be a metric space.

3. (10 points) Let V be an open subset of X and let $x \in V$. Prove that $V \setminus \{x\}$ is an open subset of X.

4. (10 points) Let $p \in X$ and $r > 0$. Prove that the closed ball $\overline{B}(p, r)$ is a closed subset of X.

5. (10 points) Show that the complement of a finite subset of X is open. That is, let $F = \{x_1, \ldots, x_k\}$ be a subset of X. Prove that the compliment $X \setminus F = \{y \in X \mid y \notin F\}$ is an open subset of X.

6. (10 points) Let $p, q \in X$ with $p \neq q$. Prove that there exists open subsets U_1 and U_2 of X such that $p \in U_1$, $q \in U_2$, and $U_1 \cap U_2 = \emptyset$.

7. (10 points) Let A be a subset of X. Suppose there exists $D > 0$ such that whenever $x, y \in A$ and $x \neq y$, then $d(x, y) \geq D$. Either prove that A is a closed subset of (X, d), or else construct a counter example showing that A need not be closed. Sometimes the set A is called the *set of guests at a bad party.*

In questions 8–9, let $(X, \mathcal{T})$ be a topological space.

8. (10 points) Prove that the intersection of any collection of closed subsets of X is closed.

9. (10 points) Suppose that A is a subset of X such that for every $x \in A$ there is an open set U containing x such that $U \subset A$. Prove that A is an open set in $(X, \mathcal{T})$.