Math 7210-01: Riemannian Geometry
MWF 1:10–2:00pm in SC 1313
Fall 2019

Instructor: Prof. Spencer Dowdall

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Office Hours:
Mon 3:00–4:00pm
Tues 1:00–2:00pm
Fri 11:00am–12:00pm
and by appointment

Course Information

Description: This course is intended for graduate students with interest in geometry (differential, algebraic, symplectic) as well as topology. It aims to cover the the main topics of Riemannian geometry: differential manifolds; tangent spaces; tangent bundles; vector fields; Riemannian connections; Riemannian metrics; geodesic flow; sectional, scalar and Ricci curvatures; Jacobi fields; complete manifolds; the Hopf-Rinow and Hadamard Theorems, and possibly the Sphere Theorem. Time permitting, we will also review complex manifolds; Hermitian and Kähler metrics; and the first Chern class.

Both conceptual and computational aspects will be emphasized. The course will provide the necessary basis for future theoretical research as well as important classes of examples that may serve as a setting for research projects and computational exploration.

Text: The required text for this course is:

Riemannian Geometry by Manfredo Perdigão do Carmo (on reserve)

We will cover most of chapters 0–8 of the text, with some omissions and some supplemental material.

Additional resources:

- *Riemannian Geometry* by S. Gallot, D. Hulin, and J. Lafontaine (on reserve)
- *A Comprehensive Introduction to Differential Geometry, vols. I,II* by M. Spivak
- *Riemannian Geometry* by P. Petersen
- *Foundations of Differential Geometry, vol. I* by S. Kobayashi and K. Nomizu
- *Morse theory* by J. Milnor

Course webpage: The course webpage is located at

`https://math.vanderbilt.edu/dowdalsd/Fa2019math7210/`

and contains all course information (including this document), announcements, homework assignments, and the current schedule of topics. Additionally, **grades for completed assignments** may be accessed by logging into the course page at `https://brightspace.vanderbilt.edu`.

Homework: Homework problems will be assigned most weeks (typically on Wednesdays) and will be due **at the beginning of lecture** the following Wednesday. Solutions should be *neat*, *legible*, and *stapled*. Students are encouraged to work together on homework assignments, but each student must write up their solutions independently and in their own words.

Projects: Each student will be required to present a short project in class during the last weeks of the term. A list of potential projects will be provided by the instructor, but you are also free to choose your own topic (with approval). The form of the project will be to present an important theorem, a class of examples, or a new concept and its main properties.

Exams: There will be no exams.

Grading: Final grades will be computed in terms of the homework grades and the final project.

Honor Code: Vanderbilt's Honor Code governs all work in this class. All work submitted for credit must be the student's own and should reflect the student's own understanding of the material.

Course plan / Tentative outline:

1. Differential Manifolds
   (a) Differential manifolds, tangent spaces
   (b) Vector fields, brackets
2. Riemannian metrics and connections
   (a) Riemannian metrics
   (b) Affine connections
   (c) Riemannian connections
3. Geodesics
   (a) Introduction
   (b) Geodesic flow
   (c) Properties of geodesics
4. Curvature
   (a) Introduction to Curvature
   (b) Sectional curvature
   (c) Ricci curvature and scalar curvature
5. Jacobi Fields
   (a) Jacobi equation
   (b) conjugate points
6. Complete manifolds
   (a) Complete manifolds
   (b) Hopf-Rinow Theorem
   (c) Hadamard Theorem
7. Complex manifolds
   (a) Introduction to complex manifolds
   (b) Hermitian metrics, Kähler metrics
   (c) Examples
   (d) First Chern class