Math 595 section CCT
Curve complexes and surface topology

TR 9:30-10:50 in Everitt 143

Fall 2014

Instructor: Spencer Dowdall

Office: Altgeld Hall 257
Office Phone: 217-244-9038
Email: dowdall@illinois.edu

Office Hours:
Tuesdays 10:50-11:50
Wednesdays 1:30–3:30
and by appointment

Course Information

Course webpage: https://www.math.uiuc.edu/~dowdall/Fa2014math595/index.html
The course webpage will contain announcements, this syllabus, any assignments/exercises, and potentially notes or other relevant info.

Grading: Course grades will assigned based on the following criteria:

- Participation – regular attendance and active engagement in lectures and course discussions.
- In-class presentations – in the latter half of the semester, we may take some time for each student to give a lecture/presentation. This could range from the proof of an important lemma to an introductory talk on a new topic. Details will be discussed later on.
- Occasional exercises – I may assign a handful of exercises over the course of the semester. For some exercises we'll choose a student present the solution in class, for other exercises the solutions will be turned in.
- There will be **no exams**.

Goals: To describe and study various "complexes of curves." These complexes encode a combinatorial relationship between curves on surfaces and have proven to be very useful tools in several areas of low-dimensional topology. We will explore the key properties of these complexes and develop some tools for working with them. Time permitting, we will discuss applications of this theory in various settings.

Tentative Outline (subject to change):

1. Intro + Definitions
   - define the curve complex, arc complex, etc...
   - discuss motivation from mapping class groups, Teichmüller space, 3–manifolds, ...
   - references: [Har3], [FM]
2. Topology
   - basic topological observations
   - profound consequences (work of Lickorish, Harer, Hatcher–Thurston, ...)
   - references: [Lic], [FM], [HT], [Hat], [Waj1],[Waj2], [Har1], [Har2].

[*] *This document was last updated **August 18, 2014**. Find the current version on the course website.*

3. Geometry – the work of Howard Masur & Yair Minsky
    - hyperbolicity [MM1], [Bow], [HPW]
    - hierarchical structure (subsurface projections, bounded geodesic image thm, Behrstock inequality, tight geodesics, . . .) [MM2]
    - The boundary and Klarreich's theorem [Kla]
4. Applications
    - distance formula for mapping class groups (Masur–Minsky [MM2])
    - distance formula for Teichmüller space (Rafi [Raf])
    - conjugacy problem for mapping class groups (Tao [Tao])
    - Thurston's Ending Lamination Conjecture (Minsky et al. [Min1, Min2, Min3], [BCM])
    - actions of Mod on product of hyperbolic spaces (Bestvina–Bromberg–Fujiwara [BBF])
    - Weil-Petersson geometry and volumes of 3–manifolds (Brock [Bro1, Bro2], Brock–Masur–Minsky [BMM1, BMM2])
5. Analogs for Out(F_n)
    - free splitting complex (Handel–Mosher) [HM]
    - free factor complex (Bestvina–Feighn) [BF]

References

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