FALL 2014

MATH 595: Curves Complexes and Surface Topology

Section CCT, CRN 62765

9:30-10:50 AM TR, 143 EVERITT

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Description: The curve complex C(S) of a surface is a simplicial complex that encodes the intersection data of all simple closed curves on the surface. This complex was introduced by Harvey in '78 and has since played a central role in the study of mapping class groups. For example, Harer famously used the curve complex (and its variants) to calculate the virtual cohomological dimension of the mapping class group and to prove homological stability for mapping class groups. Such complexes can also be used to give a simple proof that the mapping class group is finitely presented. In the past decade, the geometric and hierarchical structure of C(S) discovered by Masur and Minsky has had a transformative affect on our understanding of Teichmuller space and hyperbolic 3-manifolds.

This course will develop the theory of the curve complex and related complexes and will discuss several applications to low-dimensional topology. After discussing its topological features (connectedness, homotopy type, automorphism group), we will primarily focus on the geometry of the curve complex. Specifically, we will prove that C(S) is Gromov hyperbolic and outline the Masur-Minsky hierarchy machinery (tight geodesics, subsurface projections, bounded geodesic image theorem, and the Behrstock inequality).

Applications to surface topology will include distance formulas that coarsely measure distances in the mapping class group and Teichmuller space, and a combinatorial description of the behavior of Teichmuller geodesics. We will also discuss the role of C(S) in the proof of the ending lamination conjecture and how the pants complex relates to volumes of certain hyperbolic 3-manifolds. Time permitting, we will discuss free group analogs of the curve complex (e.g., the free factor and free splitting complexes) and their relation to the study of Out(F_n).