Let \((F, +, <)\) be a totally ordered field. A pair of subsets \(A, B \subset F\) is called gap if \(A < B\) and \(A \cup B = F\). By the theorem of Erdos, Gilman and Henriksen every two real closed fields with cardinality \(\aleph_1\), which are \(\eta_1\)-set, are order–isomorphic. G. Pestov has introduced notions of symmetric and asymmetric gaps and has proved another isomorphic theorem [1]. Let \((A, B)\) be a gap of \(F\). A set \(A\) is called short shore, if there exists \(a_0 \in A\) such that for all \(a \in A\) we have \(a + (a - a_0) \in A\). If a shore is not short then it is called long shore. If both shores \(A\) and \(B\) are long, then a gap \((A, B)\) is called symmetric. If one of shores is long and the other one is short, then such gap \((A, B)\) is called asymmetric. A gap \((A, B)\) of \(F\) is said to have \((\alpha, \beta)\)-cofinality, if \(\text{cf}(A) = \alpha\), \(\text{coi}(B) = \beta\). It is easy to see if \((A, B)\) is a symmetric gap then \(\text{cf}(A) = \text{coi}(B)\).

**Theorem [1]** Let \(F_1\) and \(F_2\) are really closed ordered fields such that \(|F_1| = |F_2| = \aleph_1\) and the cofinality of every symmetric gap in both fields is equal to \(\aleph_1\). Then \(F_1\) and \(F_2\) are isomorphic as ordered fields iff groups of archimedean classes of both fields are order–isomorphic.

Now we consider a class \(K\) of real closed fields under the following conditions
1) \(|F| = |G| = \aleph_1\), where \(G\) is a group of archimedean classes of \(F\),
2) if \((A, B)\) is a symmetric gap of \(F\) then \(\text{cf}(A, B) = \aleph_1\).

A field \(R[[G]]\) of formal power series consists of all \(x = \sum_{g \in G} r_g g\), where \(r_g \in R\), \(\text{supp}(x) = \{g \in G | r_g \neq 0\}\), \(\text{supp}(x)\) – inversely well-ordered subset of the totally ordered group \(G\). The order in \(R[[G]]\) is as follows: \(x > 0\) iff \(r_{\gamma} > 0\), where \(\gamma = \max\text{supp}(x)\). Let \(\beta\) be a cardinal, \(\aleph_0 < \beta \leq |G|\), by \(R[[G, \beta]]\) we denote a subfield of \(R[[G]]\) which consists of such formal series \(x\) that \(|\text{supp}(x)| < \beta\). We call this subfield the field of bounded formal power series.

**Theorem.** Let \((A, B)\) be a symmetric gap in \(R[[G, \beta]]\). Then \(\text{cf}(\beta) = \text{cf}(A) = \text{coi}(B)\).

We have the following facts.
1) Under CH, the class \(K\) is exactly one of all bounded formal power series \(R[[G, \aleph_1]]\), where \(G\) is a divisible abelian group, \(|G| = \aleph_1\).
2) Every symmetric gap of a field from this class has type \((\aleph_1, \aleph_1)\).
3) If \(F\) is a totally ordered real closed field, which is \(\eta_1\)-set and \(|F| = \aleph_1\), then \(F \in K\).
4) A nonstandard real line \(*R\), which is \(\eta_1\)-set, belongs to this class and it has both symmetric \((\aleph_1, \aleph_1)\) gaps and asymmetric \((\aleph_1, \aleph_1)\) gaps.
5) We consider also a construction \(R[[G(L, P), \aleph_1]]\) of fields from this class, where \(L\) is a totally ordered set, \(P\) is a totally ordered field, \(G(L, P)\) is a group of finite words with letters from \(L\) and exponents of letters from \(P\). We prove that such fields always have asymmetric \((\aleph_0, \aleph_0)\) gaps and so \(K\) is strictly greater then the class of all \(\eta_1\)-sets of cardinality \(\aleph_1\).
6) We consider also examples of fields from the class $K$ which are not $\eta_1$-sets and have an asymmetric $(\aleph_1, \aleph_1)$ gaps.

7) There exists a subfield of a nonstandard real line, which has only Dedekind symmetric gaps. The nonstandard line and its subfield both belong to the class $K$. (A gap $(A, B)$ in a field $F$ is called Dedekind gap or fundamental gap if for all strictly positive $\varepsilon \in F$ there exist $x \in A, y \in B$ such that $|y - x| < \varepsilon$.)

8) There are semi-$\eta_1 + \beta_1$-super-real fields that belong to the class $K$ and the class $K$ is wider then the class of all semi-$\eta_1$- fields of cardinality $\aleph_1$.

Questions.
1) Is every field from $K$ super-real field?
2) Does there exist a field with symmetric gaps of different cofinality?

References