Mini-courses

Discrete integrable systems and cluster algebras

P. Di Francesco; Institut de Physique Theorique, CEA Saclay, France

Abstract: We discuss discrete time non-linear evolution equations arising from the study of quantum integrable spin chains in physics, and show how they fit in the cluster algebra structure of Fomin and Zelevinsky. The latter is a sort of dynamical system on formal coordinate patches that are transformed according to rational mutation rules, guaranteeing the Laurent property that each mutated coordinate is a Laurent polynomial of the variables in any other coordinate patch. We shall focus on the main open positivity conjecture of cluster algebra, stating that these Laurent polynomials have only non-negative integer coefficients.

The combinatorial content of our evolution equations is revealed by their exact solutions, which make use of their discrete integrable structure to express them as partition functions for Viennot’s heaps of pieces, or alternatively for positively weighted paths on target graphs. Cluster algebra mutations are implemented by local transformations in various guises: local continued fraction rearrangements, matrix representations local moves a la Yang-Baxter, etc.

This generalizes nicely to a non-commutative setting, by use of the notion of quasi-determinants introduced by Gelfand and Retakh, and allows to prove some Kontsevich non-commutative cluster algebra positivity conjecture in rank 2. In higher rank, we obtain a non-commutative version of the discrete Hirota equation.

(Lectures based on work in collaboration with R. Kedem).

Free probability theory and random multi-matrix models

Dimitri Shlyakhtenko, UCLA

Abstract: We discuss certain aspects of random multi-matrix models: random matrices chosen at random according to a certain unitarily invariant Gibbs measure on the space of $N \times N$ matrices. Thanks to work of Voiculescu, Biane, Guionnet and others, free probability tools can be used to analyze the limiting behavior of such random matrix models. In addition to touching upon a connection between such random matrix models and subfactor theory, we will describe our recent joint work with A. Guionnet on free monotone transport. We will present applications both to random matrix theory and von Neumann algebra theory.
Loop groups and Chern-Simons theory
Constantin Teleman, UC Berkeley

Abstract:
Lecture 1. Review of conformal blocks, modular functors and relation to Chern-Simons form the point of view of algebraic geometry, loop groups and moduli of bundles
Lecture 2. Gauge theory in 2d, the Verlinde dimensions (Chern-Simons reduced along a circle) and twisted K-theory
Lecture 3. Chern-Simons as an extended TQFT. Anomalies, invertible TQFTs and relation to conformal nets.

Almost normal operators mod Hilbert-Schmidt and the $K$-theory of the algebras $E\Lambda(\Omega)$
Dan Voiculescu, UC Berkeley

Operator algebraic aspects of conformal field theory
Antony Wassermann; CNRS, Marseille

Abstract:
Lecture 1. Bosons and the oscillator semigroup.
Lecture 2. Fermions and singular integral operators.
Lecture 3. Vertex operator algebras
Lecture 4. Analytic aspects of vertex operators.

Semiclassical analysis, loop group characters, and the modular group action
Jonathan Weitsman, Northeastern University

Abstract: Quantized systems can display symmetry not arising from invariances of the underlying symplectic manifolds. One example of this type of "enhanced symmetry" is the appearance of modular invariance for characters of loop group representations; this group action does not arise from any known symmetry of the coadjoint orbit (though it does of course appear in conformal field theory). We show that a modular group action appears geometrically in the corresponding semiclassical category. This indicates that semiclassical analysis may make it possible to find enhanced symmetry in other situations where constructing the quantization may be difficult or unattainable.
Lecture 1: Geometric Quantization and the Weyl Character formula

Lecture 2: The symplectic category and its variants

Lecture 3: The symplectic category for quasi-Hamiltonian loop group spaces, the Kac character formula, and the modular group action.

(joint work with Victor Guillemin and Shlomo Sternberg)

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**One-hour talks**

The geometric construction of the Reshetikhin-Turaev TQFT

Jørgen Ellegaard Andersen, University of Aarhus

**Abstract:** In this talk we will discuss the geometric construction of the Reshetikhin-Turaev TQFT via conformal field theory and further via geometric quantization of moduli spaces. We will then discuss results regarding the large level asymptotics of these theories.

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Subfactors, conformal field theory and K-theory

David Evans, Cardiff University

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Existence of systems of endomorphisms of Izumi type

Terry Gannon, University of Alberta

**Abstract:** In ground breaking work, Izumi introduced a family of systems of endomorphisms containing the Haagerup at index 4.30..., and another family containing the $E_6$ subfactor. He established the existence of these systems, only for the first 2 resp. 4 members of these families. In my talk I will update my work with David Evans on existence (or otherwise) of these systems, and discuss related questions.
Quantum spin chains, correspondences and subfactors
Vaughan Jones, Vanderbilt University

Abstract: Quantum spin chains arising as the 1-dimensional quantum version of classical 2-dimensional statistical mechanical models exhibit the same Hilbert spaces as those arising by iterating correspondences in the sense of Connes. This will be explained and the highly speculative possibility of a continuum limit conformal field theory proposed.

CFT from the arc point of view and structural relations to planar algebras
Ralph Kaufmann, Purdue University

Abstract: One approach to study moduli spaces is to use arcs. Taking this approach, we defined the arc operad, which among other things models correlation functions on over a certain partial compactification of moduli space. Using a careful analysis, we then were able to restrict the theory to moduli space and to extend it to the open/closed case.

An example of these correlation functions are actions on the Hochschild cochain complex of a (Frobenius) algebra. These operations include Deligne’s conjecture, its cyclic version and string topology. These actions are built from two main ingredients, a discretization and the co-simplicial structure.

On this discretized level of actions there is a striking analogy of our theory with that of planar algebras, whose deeper connection we will elucidate and explore.

Superconformal Field Theory and Noncommutative Geometry
Yasuyuki Kawahigashi, University of Tokyo

Abstract: We will present operator algebraic approach to superconformal field theory. We emphasize representation theoretic aspects, classification theory and connections to noncommutative geometry.

Classifying subfactors: beyond index 5
Emily Peters, MIT

Abstract: The recent completion of the classification of subfactors of von Neumann algebras with index below 5 begs the question ”how far can the classification go?” While there’s evidence suggesting that extending the classification to index 6 is impossible, an upper limit of $3 + \sqrt{5} \approx 5.24$ may be possible. I’ll talk about a couple of preliminary results in this direction (uniqueness of subfactors associated to certain principal graphs), which are joint work with Scott Morrison.
Hamiltonian framework for Batalin-Vilkovisky classical field theories
Nicolai Reshetikhin, UC Berkeley

Abstract: Quantum Knizhnik-Zamolodchikov equations appear in representation theory of quantum affine universal enveloping algebras. They deform differential equations for correlation functions in WZW model in conformal field theory on a sphere. Reflections qKZ equations deform differential equations for correlation functions of the WZW model on a strip. Solutions to reflection qKZ will be constructed as Jackson multiple integrals.

Classifying all algebras in a given fusion category
Noah Snyder, Columbia University

Abstract: The goal of this talk is to explain how to classify all algebras in a given fusion category, illustrated with examples coming from my joint work with Pinhas Grossman. This question is of great interest in conformal field theory and in subfactor theory. In particular, classifying nice extensions of a conformal net reduces to understanding algebras in a fusion category. Furthermore, it allows you to construct many new subfactors from a single example, and it is crucial in understanding intermediate subfactors. The main tool in this theory is the Brauer-Picard groupoid or maximal atlas of a fusion category. In this talk I’ll concentrate on illustrating the Brauer-Picard group via examples coming both from subfactor theory and from classical algebra.

Introduction to fermionic second quantization
James Tener, UC Berkeley

Abstract: The goal of this talk is to present the basics of fermionic second quantization from an operator algebraic perspective. We will discuss the construction of fermionic Fock space from representations of the Canonical Anticommutation Relations (CAR) algebra, as well as the construction and classification of positive energy representations of loop groups and their symmetries. The majority of this talk will be given at an introductory level.

Casimir connections, generalized braid groups and affine quantum groups.
Valerio Toledano Laredo, Northeastern University

Abstract: The Kohno-Drinfeld theorem describes the monodromy of the Knizhnik-Zamolodchikov equations in terms of the R-matrix representations coming from quantum groups. I will describe analogous results for generalized braid groups. The role of the KZ equations is played here by the Casimir equations, which are flat connections on the generalized configuration spaces associated to simple Lie algebras, and that of the R-matrix by quantum Weyl group operators.
In the affine setting, this leads in particular to the construction of an explicit equivalence of categories between finite-dimensional representations of Yangians and quantum loop algebras. This talk is based in part on joint work with Sachin Gautam.

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Progress in solving noncommutative $\phi^4$-theory in four dimensions

Raimar Wulkenhaar, Westfälische Wilhelms-Universität Münster

**Abstract:** $\phi^4$-theory on 4-dimensional Moyal space with harmonic propagation is a perturbatively renormalisable quantum field theory. At the self-duality point, the model is almost scale-invariant and therefore conjectured to exist non-perturbatively. We reduce the construction of that model to the problem to prove that a certain non-linear singular integral equation for a function on the unit square has a solution. We have strong indications that, for positive coupling constants, this proof is within reach.

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On intermediate conformal nets

Feng Xu, UC Riverside

**Abstract:** Let $A$ be a completely rational net and $A \subset B$ an extension of $A$ with finite index. In this talk we will describe a recent general result which shows that the (maximal version of ) generalized Wall’s conjecture is true for the pair $A \subset B$. When combined with earlier results on cosets and conformal inclusions, this gives infinite families of subfactors from CFT which verify generalized Wall’s conjecture.

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From spintronics to operator algebras: why does skew scattering make us interested in obscure correlation functions?

Tim Ziman, Université Joseph Fourier