Hopf cyclic cohomology and transverse characteristic classes

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Abstract

Hopf cyclic cohomology originally invented by Alain Connes and Henri Moscovici in 1998 as a computational tool for computing the index cocycle appeared in local index formula. We concentrate on the recent developments and in five lectures we review Hopf algebras of transverse symmetry, their Hopf cyclic cohomology, and transverse characteristic classes.

Lecture 1 (Rangipour): Hopf Algebras

After recalling basics of Hopf algebras and reviewing the known Hopf algebras associated to Lie algebras and groups, we study Lie-Hopf algebras as a amalgamation of Lie algebras and Hopf algebras. We use Lie -Hopf algebras as the cornerstone of bicrossed product algebras. Several examples of Lie-Hopf algebras, including finite and infinite type, are explored. We recall the Hopf algebras of transverse symmetries in all possible cases and illustrate them in two more tangible instances.

Lecture 2 (Rangipour): Hopf cyclic complexes

In this lecture we introduce all complexes needed in our discussions. We start by explaining the original Hopf cyclic complex. We continue with the notion of SAYD modules and their indispensable role in the theory. We define a functor between representations of a matched pair of Lie algebras or Lie groups and the SAYD module over the corresponding Hopf algebra. We introduce the Hopf-Koszul complex as a Hopf cyclic complex with coefficients in a DG graded SAYD module. We proceed by recalling the Hopf type of Chevalley-Eilenberg complex. Eventually we define the smallest bicomplex at which the van Est isomorphism lands.

Lecture 3 (Rangipour): Characteristic maps in Hopf cyclic cohomology

We explain the need of characteristic maps as the main motivation of our study in Hopf cyclic cohomology. We associate a characteristic
map to any complex recalled in the previous lecture. We illustrate the cocycles over the crossed product algebras obtained via the action of Hopf algebras for two known cases introduced in the first lecture.

Lecture 4 (Moscovici): *The origins of Hopf cyclic cohomology*

Lecture 5 (Moscovici): *Hopf cyclic Chern classes*