The symmetry in transverse geometry comes organized in the form of certain Hopf algebras, which play a role similar to that of $\mathfrak{gl}_n$ as structure group for Hopf algebras $\mathbb{H}$, which admit a nontrivial deformation to a QDE algebra $\mathbb{H}$. Much like a classical Lie algebra deforms to a QDE algebra, the structure bundle $\mathfrak{gl}_n$ in transverse geometry comes organized in the form of certain algebras.

**Fundamental Class**

Rankin-Cohen deformations along the transverse
Each $Z \in \mathfrak{g}(u)$ acts on $\mathcal{W}_J$ as a linear transformation: 

$$\phi_{\ast \Omega}(f)Z = (\phi_{\ast \Omega}f)Z \quad \forall f \in \mathcal{W}_J$$

$$0 = [\gamma X, \gamma Y] \quad [\gamma X, \gamma \lambda] \quad [\gamma \lambda, \gamma X] - [\gamma \lambda, \gamma Y] = [\gamma \lambda, \gamma \lambda]$$

The affine extension $\mathfrak{g}(u)$ of $\mathfrak{gl}(u)$ acts on $\mathcal{W}_J(u)$, implementing the action of the standard horizontal and vertical vector fields, implementing the action of the coordinate algebra $\mathcal{A}$, consisting of finite sums of monomials of the form $\phi_{\ast \Omega}(f) = \phi_{\ast \Omega}f \quad \forall f \in \mathcal{W}_J$.

with the product given by

$$(\mathcal{W}_J)_\phi = \mathcal{W}_J \quad \phi_{\ast \Omega}f \quad \forall f \in \mathcal{W}_J$$

Each \(Z \in \mathfrak{g}(u)\) acts on \(\mathcal{W}_J\) as a linear transformation:

\[
\phi_{\ast \Omega}(f)Z = (\phi_{\ast \Omega}f)Z \quad \forall f \in \mathcal{W}_J
\]

\[
0 = [\gamma X, \gamma Y] \quad [\gamma X, \gamma \lambda] \quad [\gamma \lambda, \gamma X] - [\gamma \lambda, \gamma Y] = [\gamma \lambda, \gamma \lambda]
\]

The generators satisfy obvious product rules when acting on $\mathcal{H}$, giving a coproduct $\mathcal{H} \otimes \mathcal{H} \rightarrow \mathcal{H} : \triangleright$ compatible with the algebra structure, and satisfying all the Hopf algebra axioms.

\[
\begin{align*}
\triangleright \in \mathcal{H} & \quad \triangleright (\triangleright) \otimes (\triangleright) \triangleright = \triangleright \\
\mathcal{H} \otimes \mathcal{H} & \rightarrow \mathcal{H} : \triangleright \quad \text{giving a coproduct}
\end{align*}
\]

By multiplicativity

\[
\begin{align*}
\triangleright (\triangleright) (\triangleright) \triangleright + \triangleright (\triangleright) (\triangleright) \triangleright & = (\triangleright (\triangleright) \triangleright \\
\triangleright (\triangleright) (\triangleright) \triangleright + \triangleright (\triangleright) (\triangleright) \triangleright & = (\triangleright (\triangleright) \triangleright \\
\end{align*}
\]

The generators satisfy obvious product rules when acting on $\mathcal{H}$, giving

\[
\begin{align*}
\triangleright (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) & = \triangleright (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) \\
\triangleright (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) & = \triangleright (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright)
\end{align*}
\]

with

\[
\begin{align*}
\triangleright (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) & = \triangleright (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) (\triangleright) \\
\end{align*}
\]

The resulting Hopf algebra $\mathcal{H}$ is actually independent of the choices made, and can alternatively be described as a bicrossproduct of $\mathcal{G}$. Let $\mathcal{A}$ be the algebra of linear transformations of $\mathcal{H}$ generated by the operators $A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, Q, R, S, T, U, V, W, X, Y, Z$.
The Hopf algebra $\mathcal{H}$ is the universal enveloping algebra of the Lie algebra with basis $\{\mathfrak{h}^\mathfrak{g}\}$ and brackets

$$\mathfrak{h}^\mathfrak{g} \cdot \mathfrak{h}^\mathfrak{g} = \mathfrak{h}^\mathfrak{g}$$

The counit is determined by

$$\epsilon = (\lambda)S$$

and the property

$$\epsilon g - = (\mathfrak{h}^\mathfrak{g})S \quad \lambda^\mathfrak{g} + X^- = (\lambda)S \quad \lambda^- = (\lambda)S$$

The antipode is determined by

$$\mathcal{H} \ni \mathfrak{h}^\mathfrak{g} \cdot \mathfrak{h}^\mathfrak{g} = \mathfrak{h}^\mathfrak{g}$$

Together with the multiplicity property

$$\mathfrak{h}^\mathfrak{g} \cdot \mathfrak{h}^\mathfrak{g} = \mathfrak{h}^\mathfrak{g} \quad \lambda \cdot \mathfrak{h}^\mathfrak{g} + X \cdot \mathfrak{h}^\mathfrak{g} = X \mathfrak{h}^\mathfrak{g} \quad \lambda \cdot \mathfrak{h}^\mathfrak{g} = \lambda \mathfrak{h}^\mathfrak{g}$$

As coalgebras,

$$0 = [\mathfrak{h}^\mathfrak{g}, \mathfrak{h}^\mathfrak{g}] \quad I + \mathfrak{h}^\mathfrak{g} = [\mathfrak{h}^\mathfrak{g} \cdot X] \quad \mathfrak{u} \cdot \mathfrak{h}^\mathfrak{g} = [\mathfrak{h}^\mathfrak{g} \cdot \lambda] \quad \lambda = [X \cdot \lambda]$$

and brackets

$$\{I \leq \mathfrak{u}, \mathfrak{h}^\mathfrak{g} \cdot \lambda, X\}$$

As algebra $\mathcal{H}$ is the universal enveloping algebra of the Lie algebra with basis $\mathfrak{h}^\mathfrak{g}$ and brackets

$$\mathfrak{h}^\mathfrak{g} \cdot \mathfrak{h}^\mathfrak{g} = \mathfrak{h}^\mathfrak{g}$$

The Hopf algebra $\mathcal{H}$ is
Given a \( ^{1}M \) of dimension \( 1 \), but the twisted antipode does not satisfy \( ^{1}\mathcal{H} \otimes \otimes^{1}\mathcal{H} \). The twisted antipode is defined as \( \delta \mathcal{A} \) on the crossed product algebra \( \mathcal{T} = \mathbb{R} \times \mathcal{W} \simeq (^{1}\mathcal{W})^{+} \). Given an \( \mathbb{R} \) manifold \( \mathcal{Y} \) and a subgroup \( \mathbb{D} \subseteq \text{Diff}_{+}(^{1}\mathcal{W}) \), the twisted antipode acts on the crossed product algebra \( \mathcal{T} = \mathbb{R} \times \mathcal{W} \simeq (^{1}\mathcal{W})^{+} \).
Thus, the effective action on $\mathcal{A}$ is that of the quotient Hopf algebra

$$\frac{\mathcal{A}}{\mathcal{H}} = \mathcal{L}$$

by the ideal and coideal – generated by the primitive element $\mathcal{O}_i \in \mathcal{H}$.

Thus, the effective action on $\mathcal{A}$ is that of the quotient Hopf algebra

$$\mathcal{A} \otimes ((\mathcal{H}_i D)_{i_f})_\infty ^c \mathcal{O} = \mathcal{A} \otimes \mathcal{H} \ni \mathcal{O}_i \mathcal{O}_i = \mathcal{O}_i$$

acts as

$$0 \equiv \left( \left( \frac{xp}{\phi p} \mathcal{O}_i \right) \frac{xp}{p} \right) \mathcal{O}_i - \left( \frac{xp}{\phi p} \mathcal{O}_i \right) \frac{xp}{\phi p} =: \{ x : \phi \}$$

linear fractional transformations. One has $\phi \in \mathcal{L} \Leftrightarrow \phi$ is Schwarzian derivative.

In particular take projective (i.e. $PSL(2,\mathbb{R})_\infty \mathcal{A} = \mathcal{L}$).
\[
(1 \times \left( ((1_W)^+ f) \otimes O \right)_{\text{V}})_{\omega} \in \text{PF cyclic classes of } PH \circledast \text{Fundamental classes of } O
\]

Last but not least \( \chi \) is a Hopf cyclic \( \omega \)-cocycle, \( \lambda \otimes \lambda \omega = \lambda \otimes \lambda - \lambda \otimes \lambda = 0 \).

Thus \( (1) H \mapsto H \in PH \circledast \text{Fundamental classes of } O \)

\[
\int \left( \frac{1}{x} f \cdot \left( ((1_W)^+ f) \otimes O \right)_{\text{V}} \cdot \left( (1_x)^+ (1_W)^+ f \right) \right) = \left( (1^* \Omega 1 f)^0 \cdot \star \Omega 0 f \right)^{\star \chi} = \left( (1^* \Omega 1 f)^0 \cdot \star \Omega 0 f \right)^{\star \chi}
\]

Into the Godbillon-Vey class:

\[
(1_x) \otimes \left( \left( (1_x)^+ f \right) \otimes O \right)_{\omega} = \left( (1_x) \otimes \cdots \right) \otimes \left( (1_x) \right)_{\omega}^{\star \chi}
\]

\[
\text{which is mapped by the characteristic homomorphism}
\]

\[
\left( \left( 1 \otimes \left( \left( 1_W \right)^+ f \right) \otimes O \right)_{\omega} \right)_{\omega} = \left( 1 \otimes \omega \right)_{\omega} = \left( 1 \otimes \omega \right)_{\omega} = \left( 1 \right)_{\omega}^{\star \chi} = \left( 1 \right)_{\omega}^{\star \chi}
\]

Thus a cyclic cocycle, hence a cyclic cocycle. One has

\[
\left( 1 \otimes \left( \left( 1_W \right)^+ f \right) \otimes O \right)_{\omega} \in [\omega] \text{ gives a class }
\]

Which cyclic classes
A deformation of a Hopf algebra is a Hopf algebra structure on the topological module \(\mathcal{H}\) over the ring \(R\), such that

\[
\Delta^2 + \gamma^2 + \alpha + \beta = 0
\]

Moreover, if \(R_{F_{[2]}^{-1}}\) satisfies \(\text{QYBE}\), then

\[
R_{F_{[2]}^{-1}}(2) = R_{F_{[2]}^{-1}}[i, j, k] + [j, k, l] + [j, k, o]
\]

If and only if \(R\) satisfies the classical Yang-Baxter equation

\[
\Delta = \Delta^2 + \gamma^2 + \alpha + \beta
\]

with

\[
\gamma = \gamma^2 + \alpha + \beta
\]

Example: QVE algebras [Drinfeld, 1983]. Let \(\mathcal{H}\) be a finite-dimensional real Lie algebra, and let \(\mathfrak{g} \in \mathfrak{g} \otimes \mathfrak{g}\) be skew-symmetric. There exists a deformation

[\[\mathcal{H}\]]_{[\mathfrak{g}]} of \([\mathfrak{g}]\) given by a twisting of the form

\[
\mu = \mu^2 + \mu^3 + \mu^4 + \mu^5 + \mu^6 + \mu^7 + \mu^8 + \mu^9 + \mu^{10}
\]

Moreover, let \(\mathcal{H}\) be an invertible element such that

\[
\mathcal{H} \otimes \mathcal{H} = \mathcal{H} \otimes \mathcal{H}
\]

and

\[
\mathcal{H} \otimes \mathcal{H} = \mathcal{H} \otimes \mathcal{H}
\]

Twisting a Hopf algebra into a Hopf algebra \([\mathfrak{g}]\mathcal{H}\) with \(\mathcal{H} = \mathcal{H} \otimes \mathcal{H}\) satisfies QYBE

\[
\mu = \mu^2 + \mu^3 + \mu^4 + \mu^5 + \mu^6 + \mu^7 + \mu^8 + \mu^9 + \mu^{10}
\]

and

\[
\mathcal{H} \otimes \mathcal{H} = \mathcal{H} \otimes \mathcal{H}
\]

such that

\[
\mathcal{H} = \mathcal{H} \otimes \mathcal{H}
\]

Definition. A deformation of a Hopf algebra \(\mathcal{H}\) is a Hopf algebra structure on the topological module \(\mathcal{H}\) over the ring \(\mathcal{C}\), such that

Deformations of Hopf Algebras by Twisting
Given a Hopf algebra $H$, a necessary condition for $\mathcal{H} \otimes \mathcal{H} \in \mathcal{H}$ to define the R-C QUANTIZED Hopf algebra is:

$$(\mathcal{H} \otimes \mathcal{H})(\mathcal{H} \otimes \mathcal{H}) = \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H}$$

To illustrate its degree of complexity, here is the 3rd component:

$$\mathcal{H} [\Psi] \otimes \mathcal{H} \in \mathcal{H}$$

The series $\mathcal{T} \mathcal{H} \in \mathcal{T} \mathcal{H}$ as:

$$\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} = \mathcal{T} \mathcal{H}$$

which is precisely the same as:

$$\mathcal{H} \otimes I + (\mathcal{H} \otimes \mathcal{H}) = \mathcal{H} \otimes \mathcal{H}$$

is that $\mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} \otimes \mathcal{H} = \mathcal{T} \mathcal{H}$
The proof of Theorem A involves the framework of modular forms.

\[ \sum_{u=s+1}^{u=t} \beta_{(u)} \left( I - \gamma + u \right) \left( I - \delta + u \right)^{u-t} =: \beta_{(t)} \left( \sum_{u=s+1}^{u=t} \beta_{(u)} \right) \]

Assume \( \gamma \in (q \otimes \nu)_A \). Then any twisting element induces an associative deformation of \( \mathcal{H} \).
by requiring that the determinant belongs to $\mathcal{O} \equiv \mathbb{Q} \cap \mathbb{A}$. A modular form of weight $\omega$ is a holomorphic function $f$ satisfying

$$f\big|_\omega \mathbb{Z} \equiv \omega \big| f \quad \text{for all } \omega \in \mathcal{O}.$$ 

Equivalently, $\omega \mathbb{Z} \subsetneq \mathbb{Q} \mathbb{Z}$ where $\mathbb{Q} \mathbb{Z}$ is the group of all integral multiples of $\mathbb{Q}$.

A richer algebra emerges when besides the modular group, one considers its principal congruence subgroups and the projective limit of Riemann surfaces.

$$\lim_{\mathbb{P} \to 0} \mathbb{P} \mathbb{Z} \equiv \mathbb{Q} \mathbb{Z}.$$ 

A graded algebra emerges which is also holomorphic at $\infty$, i.e., at $z = \infty$.

$$f \big|_{z = \infty} = \left( \frac{p+zc}{q+zd} \right)_{z = \infty} f \big|_{z = \infty} = (z)^{a+b} f \big|_{z = \infty},$$

where $a, b \in \mathbb{Z}$ and the subgroup $\mathbb{P} \mathbb{Z}$ is defined by requiring that the determinant belongs to $\mathcal{O} \equiv \mathbb{Q} \cap \mathbb{A}$.

1 Modular forms and actions of \( \mathbb{H} \).
Foreach \( \nu \), one has a graded algebra of forms of level \( \nu \), and \( \wp \) is the discriminant.

\[
\wp(z) = b \prod_{i=1}^{\infty} b(z^{i}) = (z)^{2} \nabla
\]

\[
\lambda \cdot (\wp \otimes \nu) \frac{zp}{p} - \frac{zp}{p} \frac{\nu z}{1} = \lambda \cdot (\nabla \otimes \nu) \frac{zp}{p} - \frac{zp}{p} \frac{\nu z}{1} = \lambda
\]

Most natural action. First, let \( \lambda \) act on \( \nu \) as the Ramanujan operator.

bundle of \( S \) by discrete subgroups of \( D \).

There are natural actions of the Hopf algebras on the crossed products of the polynomial functions on the frame \( \mathcal{H} \), on the crossed products of the polynomial functions on the frame \( \mathcal{H} \), and analogous to the

\[
\wp \in \mathcal{H}, \quad \nu \in \mathcal{V}, \quad \wp \nabla \in \mathcal{V}
\]

with the product given by:

\[
\wp \cdot \wp \in \mathcal{H}, \quad \wp \cdot \wp \in \mathcal{V}
\]

consisting of finite sums of symbols of the form \( \wp \).

One can then form the crossed product algebra.

\[
((\mathcal{N} \ast \mathcal{V})_{\mathfrak{G}} \xrightarrow{\lim} \mathcal{V})_{\mathcal{N}} \rightarrow \mathcal{V}_{\mathcal{N}} \]

For each \( \mathcal{N} \), one has a graded algebra of forms of al levels. They define the algebra of modular forms of all levels. The group \( \mathcal{G} \) acts, sideways, on the tower defining the price.
Secondly, let
\[ \Delta \in \mathcal{H} \] act on \( \lambda \) as the grading operator.

For any \( \lambda \in \mathcal{H} \) and \( \Delta \in \mathcal{G} \), one has
\[ \lambda \cdot \Delta = (\lambda) \quad \text{where} \quad (\lambda) \Delta (\lambda) = (\lambda) \Delta (\lambda) \]

Proposition. There is a unique Hopf action of \( \mathcal{H} \) on \( \mathcal{G} \) determined by
\[ \Delta \quad \text{is the holomorphic (but not modular) Eisenstein series of weight 2. One can show} \]
\[ \frac{\zeta(u + z \omega)}{1} \sum_{\gamma \in \mathcal{G}} \sum_{p \leq u} \zeta \gamma \gamma = (\zeta) \gamma \gamma \]

where
\[ \frac{\nabla}{\zeta} \| \frac{z \rho}{p} \| \frac{1}{\gamma} \]

Equivalently, for any \( \lambda \in \mathcal{H} \) and \( \Delta \in \mathcal{G} \), one has
\[ \lambda \in \mathcal{H} \quad \text{and} \quad \Delta \in \mathcal{G} \quad \text{where} \quad \lambda \Delta (\lambda) = (\lambda) \Delta (\lambda) \]
Remark. The Schwarzian cocycle acts as inner derivation, namely, there is no choice of \( \varphi \) for which

\[
0 \equiv \frac{d}{dx} \left( \varphi \frac{d}{dx} + \left( \lambda \right) \right) = \varphi \cdot \frac{d}{dx}
\]
as and the Schwarzian cocycle acts on \( \lambda \). Then \( \lambda = \lambda(\varphi) \) and \( (\lambda) \) acts on \( \varphi \) such that

\[
0 = (\varphi) \quad \text{such that} \quad \left( (\lambda) + \varphi \right) Z \in \mathcal{A} \mathcal{W} \quad \forall \mathcal{A} \mathcal{H} \subset \mathcal{A} \mathcal{W}
\]

Proposition. \( \left( (\lambda) \right) \mathcal{H} = \mathcal{W} \mathcal{H} \equiv \mathcal{A} \mathcal{W} \quad \forall \mathcal{A} \mathcal{H} \supset \mathcal{W} \mathcal{H} \), such that \( \mathcal{A} \mathcal{W} \), \( \mathcal{A} \mathcal{W} \mathcal{H} \), and \( \mathcal{A} \mathcal{W} \mathcal{H} \mathcal{W} \) form an invertible element

\[
\cdot \left( (\lambda) \mathcal{H} \right) \mathcal{W} \mathcal{H} = \left( (\lambda) \mathcal{H} \right) \mathcal{W} \mathcal{H} \mathcal{W}
\]

The perturbed action on \( \mathcal{A} \mathcal{H} \) of \( \mathcal{W} \mathcal{H} \mathcal{W} \) is an invertible element of the convolution algebra of linear maps \( \mathcal{H} \mathcal{W} \mathcal{H} \).

Perturbations by \( \lambda \)-cocycles. \( \mathcal{H} \)-cocycle acts as inner derivation,

\[
\cdot \left[ \lambda \right] = \left[ \lambda \right] \quad \forall \mathcal{A} \mathcal{H} \subset \mathcal{A} \mathcal{W}
\]

Implemented by the weight modular form \( \lambda \mathcal{H} \).

Remark. The Schwarzian cocycle acts as inner derivation,
The normal order form of \( B \) suggests that the higher Rankin-Cohen brackets for an action of \( \mathcal{H} \) on a graded algebra \( \mathcal{A} \), such that 

\[ (u X) S \]

such that \( \mathcal{A} \) satisfies the higher Rankin-Cohen brackets for an action of \( \mathcal{H} \) on \( \mathcal{A} \), suggests that the higher Rankin-Cohen brackets for an action of \( \mathcal{H} \) on \( \mathcal{A} \), such that 

\[ (u X) S \]

should be of the form: 

\[ \frac{\mathcal{C}}{u} = \mathcal{C} | \lambda \]

However, these are not stable under \( \mathbb{Z}_1 \)-cocycle perturbations. E.g., by direct computation: 

\[ (q \lambda \cup (p) (1 + \lambda \zeta) \lambda - (q)(1 + \lambda \zeta) \lambda \cup (p) \lambda - (q) \lambda \cup (p)(1 + \lambda \zeta) \lambda + (q)(1 + \lambda \zeta) \lambda (p)(1 + \lambda \zeta) (X) S + (q)(1 + \lambda \zeta) \lambda (p) (X) S =: (q \psi (p)) \mathcal{C} \]

with 

\[ \lambda \]

obtained from recurrence relations. 

\[ (q \lambda \cup (p)(1 + \lambda \zeta) \lambda - (q)(1 + \lambda \zeta) \lambda \cup (p) \lambda - (q) \lambda \cup (p)(1 + \lambda \zeta) \lambda + (q)(1 + \lambda \zeta) \lambda (p)(1 + \lambda \zeta) (X) S + (q)(1 + \lambda \zeta) \lambda (p)(X) S =: (q \psi (p)) \mathcal{C} \]

The stable formula turns out to be of the form: 

\[ (q \psi (p)) \mathcal{C} \]

with 

\[ \mathcal{C} \]

obtained from recurrence relations. 

The following 'naturality' property of the resulting RC-brackets plays a crucial role:

\[ (q \psi (p)) \mathcal{C} \]

However, these are not stable under \( \mathbb{Z}_1 \)-cocycle perturbations. E.g., by direct computation: 

\[ (q \lambda \cup (p)(1 + \lambda \zeta) \lambda - (q)(1 + \lambda \zeta) \lambda \cup (p) \lambda - (q) \lambda \cup (p)(1 + \lambda \zeta) \lambda + (q)(1 + \lambda \zeta) \lambda (p)(1 + \lambda \zeta) (X) S + (q)(1 + \lambda \zeta) \lambda (p)(X) S =: (q \psi (p)) \mathcal{C} \]

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\[ \mathcal{C} \]

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with 

\[ \mathcal{C} \]

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with 

\[ \mathcal{C} \]

obtained from recurrence relations. 

The following 'naturality' property of the resulting RC-brackets plays a crucial role:

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However, these are not stable under \( \mathbb{Z}_1 \)-cocycle perturbations. E.g., by direct computation: 

\[ (q \lambda \cup (p)(1 + \lambda \zeta) \lambda - (q)(1 + \lambda \zeta) \lambda \cup (p) \lambda - (q) \lambda \cup (p)(1 + \lambda \zeta) \lambda + (q)(1 + \lambda \zeta) \lambda (p)(1 + \lambda \zeta) (X) S + (q)(1 + \lambda \zeta) \lambda (p)(X) S =: (q \psi (p)) \mathcal{C} \]
Lemma. Let $H$ act on an algebra $A$ with $G$-invariant inner, and let $u \in A$ be invertible.

Theorem B. The following $*$-product defines an associative deformation of $A$ and $H$:

\[ u (q \cdot a) \mapsto (q \cdot a)^u A \]
\[ \tilde{T} \otimes \mathbb{1} + \mathbb{1} \otimes \tilde{T} = (\tilde{T}) \nabla \]

The element \( \tilde{\mathcal{H}} \in (\tilde{\mathcal{O}}) \tilde{\mathcal{G}}' + (\tilde{\mathcal{O}}) \tilde{\mathcal{G}} - \tilde{\mathcal{G}} =: \tilde{T} \)

\( \mathcal{O} \times (\tilde{\mathcal{O}}) \mathcal{G} \subset \mathcal{G} \otimes \mathcal{G} \otimes \mathcal{G} \).

Next form the algebra \( \tilde{\mathcal{H}} \) as a \( \mathcal{G} \)-module with left and right action

\[ \mathcal{O} \times \mathcal{G} \times \mathcal{G} \times \mathcal{G} \]

\( \mathcal{O} \otimes \mathcal{G} \otimes \mathcal{G} \).
Let $Q$, $Q'$, $T$, $T'$ be maps of bimodules defined by $U$, $V$. 

In particular, each RC-bracket uniquely determines an element $\mathcal{R} \subset \mathcal{H} \mathcal{R} \subset \mathcal{H} \mathcal{R}$. Moreover, by Theorem B one gets:

Ellipticity Lemma. For each $n \in \mathbb{N}$, one has

$$\mathcal{E} \mathcal{H} \mathcal{R} \subset \mathcal{H} \mathcal{R} \subset \mathcal{H} \mathcal{R}$$

be the map of $d$, $d'$-bimodules defined by

$$\left((\mathcal{O} + \mathcal{C}) \mathcal{T} \mathcal{O} \times \cdots \times (\mathcal{O} + \mathcal{C}) \mathcal{T} \mathcal{O}\right)_{\text{Hom}} \left(\mathcal{H} \mathcal{R} \subset \mathcal{H} \mathcal{R} \subset \mathcal{H} \mathcal{R}\right).$$

The proof of Theorem A follows by specializing $\mathcal{E} \mathcal{H} \mathcal{R}$ to $D$. i.e. sending $\mathcal{E} \mathcal{H} \mathcal{R} \mathcal{R} \mathcal{H} \mathcal{R} \mathcal{R} \mathcal{R}$ onto the Hopf-algebraic homomorphism $\mathcal{H} \mathcal{R} \mathcal{R} \mathcal{H} \mathcal{R} \mathcal{R} \mathcal{R} \mathcal{R} \mathcal{R}$ via the perturbation proposition.
We end by recording the most significant consequence of Theorem C.