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Polarization problems
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In this talk, I will give a survey of recent developments about the polarization problems. The strong polarization conjecture asserts that for any given system of $n$ unit vectors $u_1, \ldots, u_n$ in $\mathbb{R}^n$, there exists another unit vector $v$ so that the sum of the terms $1/(u_i, v)^2$ is at most $n^2$. I will present several analytic and algebraic generalizations as well as numerous partial results. The topic is deeply connected with the theory of homogeneous polynomials, energy minimizing systems, tight frames, and the notion of inverse eigenvectors.

Approximation of Functions on a Complement of a John Domain
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We discuss Jackson-Mergelyan type theorems on the uniform approximation of continuous functions on a set “without cusps on the boundary that point inside of the set”. In particular, we derive the results on approximation of a function by reciprocals of complex polynomials which are analogues to a remarkable theorem by Levin and Saff on a real approximation. The function is continuous on a quasismooth (in the sense of Lavrentiev) arc in the complex plane. We also discuss a harmonic counterpart of a result by Mezhevich and Shirokov on the analytic polynomial approximation of continuous functions on a set consisting of two parallel segments.

On a problem by Steklov
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Coauthors: Sergei Denisov and Dmitri Tulyakov

One version of the Steklov’s problem is to obtain the bounds on the sequence of polynomials $\{P_n(x)\}_{n=0}^\infty$, which are orthonormal with respect to the strictly positive weight $\rho$. In 1921, V.A. Steklov made a conjecture that a sequence $\{P_n(x)\}$ is bounded at any point $x \in (-1, 1)$, i.e.,

$$\limsup_{n \to \infty} |P_n(x)| < \infty$$
provided that the weight $\rho$ does not vanish on $[-1,1]$. In 1979, Rakhmanov disproved this conjecture by constructing a weight from the Steklov class, for which
\[
\limsup_{n \to \infty} |P_n(0)| = \infty.
\] (1)

The Rakhmanov’s counterexample was obtained as a corollary of the corresponding result for the polynomials $\{\phi_n\}$ orthonormal on the unit circle with respect to measures from the Steklov class $S_\delta$ defined as the class of probability measures $\sigma$ on the unit circle satisfying $\sigma' \geq \delta/(2\pi)$ at every Lebesgue point. The version of Steklov’s conjecture for this situation would be to prove that the sequence $\{\phi_n(z,\sigma)\}$ is bounded in $n$ at every $z \in T$ provided that $\sigma \in S_\delta$.

In the proof of (1), an important role was played by the following extremal problem. For a fixed $n$, define
\[
M_{n,\delta} = \sup_{\sigma \in S_\delta} \|\phi_n(z,\sigma)\|_{L^\infty(T)}.
\]
Thus (1) was a corollary of the lower bound proven by Rakhmanov
\[C \ln n \leq M_{n,\delta}, \quad C > 0.\]

We recall here a well-known estimate from above:
\[M_{n,\delta} \leq \sqrt{n + 1/\delta}, \quad n \in \mathbb{N}.
\]
Later in 1981 Rakhmanov has improved his lower bound
\[C \sqrt{n \ln^3 n} \leq M_{n,\delta}, \quad C > 0.\] (2)

In our joint paper with Sergei Denisov and Dmitri Tulyakov (see [1]) we obtain the sharp bounds for the problem of Steklov, i.e. the problem of estimating the growth of $\phi_n$. We get rid of the logarithmic factor in the denominator in (2) and thus prove the optimal inequalities
\[C(\delta) \sqrt{n} < M_{n,\delta} \leq \sqrt{n + 1/\delta}.
\]

We shall discuss our approach and corresponding results.

Computing recurrence coefficients of multiple orthogonal polynomials
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Multiple orthogonal polynomials satisfy a number of recurrence relations, in particular there is a \((r + 2)\)-term recurrence relation connecting the type II multiple orthogonal polynomials near the diagonal (the so-called stepline recurrence relation) and there is a system of \(r\) recurrence relations connecting the nearest neighbors (the so-called nearest neighbor recurrence relations). In this talk we deal with two problems. First we show how one can obtain the nearest neighbor recurrence coefficients (and in particular the recurrence coefficients of the orthogonal polynomials for each of the defining measures) from the stepline recurrence coefficients. Secondly we show how one can compute the stepline recurrence coefficients from the recurrence coefficients of the orthogonal polynomials of each of the measures defining the multiple orthogonality.

A greedy algorithm for selecting piecewise polynomial wavelets centered on a non-uniform knot sequence.
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Let \(a\) be a finite knot sequence on a closed interval \([\mu, \nu]\), including the endpoints. For each knot \(a < \nu\), we choose a breakpoint \(b_a\) between \(a\) and its successor. For \(a < \nu\) we construct a finite set of functions \(\Phi_a\). \(\bigcup_{a < \nu} \Phi_a\) is an orthonormal basis consisting of continuous compactly supported piecewise quadratic functions defined on \([\mu, \nu]\) with breakpoints in the set \(a \cup \{b_a \mid a < \nu\}\). If \(\mu < a_1 < a < a_2 < \nu\) and \(a_1, a, a_2\) are consecutive knots, then \(\Phi_a\) consists of three functions: one with support \([a_1, a_2]\) and the other two with support \([a, a_2]\). \(S_a\), the linear span of \(\bigcup_{a < \nu} \Phi_a\), contains the spline space \(S_2^0(\mu, \nu)\).

Suppose \(a_1 < a_2 < a < a_3 < a_4\) are consecutive knots. If we remove \(a\) from \(a\), the resulting knot sequence \(a^1\) has the following property: keeping the original breakpoints for knots \(k < a_2\) and \(k \geq a_3\), and letting \(a\) be the breakpoint for \(a_2\) relative to \(a^1\), we have \(S_a \subset S_{a^1}\). \(S_a \cap S_{a^1}\) is spanned by three orthonormal “wavelets”, one with support \([a_1, a_3]\), one with support \([a_2, a_3]\), and one with support \([a_2, a_4]\). Let \(\Psi_a\) be the span of these three wavelets.

Our greedy algorithm starts with a knot sequence \(a\) and a function \(f\) on \([\mu, \nu]\). We find \(a\) so that the norm of the projection of \(f\) onto the span of \(\Psi_a\) is minimized. We then remove \(a\) from \(a\), as described above, to get knot sequence \(a^1\). We continue this knot removal process, generating a sequence of knot sequences \(a \supset a^1 \supset \cdots \supset a^N\) with \(S_a \supset S_{a^1} \supset \cdots \supset S_{a^N}\) so that \(N\) is
the largest index with the property that the norm of the projection of \( f \) onto \( S_n \cap S_{n/2} \) is below a prescribed threshold. The projection of \( f \) onto \( S_{n/2} \) is a good approximation of its projection onto \( S_n \). We conclude with an application to image data.

Extensions of boundary mappings
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Let \( \Omega \) be a closed bounded simply-connected region in \( \mathbb{R}^d \), \( d \geq 2 \). Consider a given mapping \( \varphi \) from the unit sphere \( S^{d-1} \) in \( \mathbb{R}^d \) to the boundary of \( \Omega \). The problem is to construct a well-behaved extension mapping \( \Phi \) from the unit ball \( B^d \) in \( \mathbb{R}^d \) to \( \Omega \), with \( \Phi = \varphi \) when restricted to \( S^{d-1} \). Consider the special case that \( \Omega \) is convex and has a smooth boundary. We give an explicit formula that involves both interpolation of the mapping \( \varphi \) and integration over \( S^{d-1} \). The integral over \( S^{d-1} \) must be done numerically. The mapping \( \Phi \) is approximated further with a truncated Fourier expansion using orthonormal polynomials over \( B^d \). We conclude with some examples for \( d = 2 \) and \( d = 3 \).

Inverse Potential Problems and higher dimensional Plemelj formulas
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We consider inverse problems in \( \mathbb{R}^3 \) for Newton potentials with density in divergence form. These arise for instance in inverse magnetization problems (e.g. for Geosciences), or in inverse source problems (e.g. for medical applications to EEG or MEG). A basic issue is nonuniqueness, that is, how to describe densities which produce zero field off the support. When the density has 2-dimensional support, we shall give such a description in terms of a decomposition of 3-D vector fields on a 2-D surface, that we call Hardy-Hodge decomposition, which generalizes the Hodge decomposition for tangential vector fields and can be seen as a higher-dimensional analog of Plemelj formulas in Clifford analysis.
Tail Median Estimation vs Tail Mean Estimation: Evidence from the Exponential Power Distribution
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We will use the asymptotic properties of the incomplete gamma function to prove which is more preferable in the limit, the median or the mean estimation.

Faber polynomials of matrices for non-convex sets
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It has been recently shown that $\|F_n(A)\|_2$, where $A$ is a linear continuous operator acting in a Hilbert space, and $F_n$ is the Faber polynomial of degree $n$ corresponding to some convex compact $E \subset \mathbb{C}$ containing the numerical range of $A$. Such an inequality is useful in numerical linear algebra, it allows for instance to derive error bounds for Krylov subspace methods. In the present paper we extend this result to not necessary convex sets $E$.

The signed equilibrium problem with log concave density
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We consider the equilibrium measure on a subset $K$ of the real line with given external field. This is a probability measure on $K$ minimizing the energy which is a useful tool in approximation theory. The signed equilibrium measure is a signed measure giving constant potential on $K$. Let $S$ be the support of the equilibrium measure and let $[a,b]$ be a subinterval of $K$. We show that if the signed equilibrium measure has log concave positive density on $[a,b]$, then $S \cap [a,b]$ is an interval.
Asymptotics and stability of critical points of Riesz potential
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We study the asymptotic behavior of the critical points of the Riesz potential $1/r^{2\beta}$, generated by the system of positive unit point charges placed at the vertices of a regular polygon, with respect to the number of charges $n$ and the Riesz parameter $\beta$. We also show that for the values of the Riesz parameter $\beta$ in a small neighborhood of $\beta = 1$, the Riesz potential $1/r^{2\beta}$ has only one equilibrium point different from the origin on each perpendicular bisector, and one equilibrium point at the origin.

Convergence in Capacity of Rational Approximants for Meromorphic Functions
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We investigate the growth and the distribution of rational approximants $r_{n,m_n}$ with numerator degree $\leq n$ and denominator degree $\leq m_n$ for meromorphic functions $f$ on a compact set $E$ of $\mathbb{C}$ where $m_n = o(n)$ as $n \to \infty$. We show that a geometric rate of convergence of $r_{n,m_n}$ to $f$ implies convergence in capacity to $f$ outside of $E$. Furthermore, we show that the convergence in capacity is uniform at least for a subsequence of $\{r_{n,m_n}\}_{n \in \mathbb{N}}$. Especially, for best rational approximants we obtain maximally convergent approximations in capacity. Moreover, we indicate applications for the distribution of the zeros of $r_{n,m_n}$ of Jentzsch-Szegö type in the case that $f$ has a branch point on the boundary of the maximal Green domain $E_\rho(f)$ of meromorphy. As an essential tool we use a characterization of the convergence rate for analytic function in the neighborhood of an isolated singular point, due to Gonchar.

Abelian integrals without quadratures
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We propose a function theoretical method for calculation (with the machine accuracy) of abelian integrals without usage of quadrature formulas. The method grounds on formulas which go back to B. Riemann and it may be applied e.g. to computation of conformal mappings of (rectangular) polygons.

On Borsuk’s conjecture  
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A new result on Borsuk’s conjecture will be presented. Namely we answer Larman’s question on Borsuk’s conjecture for two-distance sets. We find a two-distance set consisting of 416 points on the unit sphere in dimension 65 which cannot be partitioned into 83 parts of smaller diameter. This also reduces the smallest dimension in which Borsuk’s conjecture is known to be false. Other examples of two-distance sets with large Borsuk numbers are given.

Pointwise optimal multivariate spline method for recovery of twice differentiable functions on a simplex  
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Let $T \subset \mathbb{R}^d$, $d > 1$, be a non-degenerate simplex whose every face (of any dimension) contains its circumcenter. Let $W^2 T$ be the class of functions $f$ that are $C^1$-continuous on $T$ and have uniformly bounded second order derivatives in any direction. We construct an optimal algorithm on the class $W^2 T$ (in the worst-case error sense) for recovery of the function value $f(x)$ at an arbitrary given point $x \in T$ based on the information given by the values and gradients of $f$ at the vertices of $T$.

The optimal method is a linear-bi-linear spline over a certain polygonal partition of $T$ related to the Voronoi diagram of the vertices of $T$. The error $\mathcal{E}(x)$ on the class $W^2 T$ of the optimal algorithm for recovery of $f(x)$ is a $C^1$-continuous quadratic spline over the same partition of $T$. The function $\mathcal{E}$ belongs to the class $W^2 T$ and can be considered as a multivariate analogue of the classical Euler spline $\varphi_2$.

In the case $d = 1$ this result was obtained by Bojanov in 1975. An optimal algorithm for recovery of $f(x)$ on the class $W^1 T$ (and also on the class of Hölder-continuous functions on $T$) based on the values of $f$ at the vertices of $T$ was found by Babenko in the 1970s. The problem of optimal recovery of $f(x)$ on the class $W^2 T$ based on the information given only by the values of $f$ at the vertices of $T$ was solved by Kilizhekov in 1996 who showed the optimality of the linear interpolating polynomial.
Determining Singularities Using Row Sequences of Padé-orthogonal Approximants
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We study the relation of the convergence of poles of row sequences of Padé-orthogonal approximants (Padé approximants of orthogonal expansions) and the singularities of the approximated function. We prove both direct and inverse results for these row sequences. Thereby, we obtain analogues of the theorems of R. de Montessus de Ballore and E. Fabry.

Polynomial techniques for investigation of spherical designs
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The spherical designs were introduced in 1977 by Delsarte, Goethals and Seidel as a counterpart on the Euclidean sphere of the classical combinatorial designs.

A spherical $\tau$-design is a finite set $C \subset S^{n-1}$ such that the equality

$$\frac{1}{\mu(S^{n-1})} \int_{S^{n-1}} f(x) d\mu(x) = \frac{1}{|C|} \sum_{x \in C} f(x)$$

(where $\mu(x)$ is the usual Lebesgue measure) holds for all polynomials $f(x) = f(x_1, x_2, \ldots, x_n)$ of degree at most $\tau$ (i.e. the average of $f$ over the set is equal to the average of $f$ over $S^{n-1}$).

An equivalent definition says that $C \subset S^{n-1}$ is a spherical $\tau$-design if and only if

$$\sum_{x \in W} f(\langle x, y \rangle) = |C| f_0$$

holds for every $y \in S^{n-1}$ and every real polynomial $f(t)$ of degree at most $\tau$, where $f_0$ is the first coefficient in the Gegenbauer expansion of $f(t) = \sum_{i=0}^{k} f_i P_i^{(n)}(t)$.

In this talk we show how (1) can be used for some special (with respect to the the design) points implying restrictions on the structure of the designs. This idea was firstly proposed and used by Fazekas-Levenshtein (1997) and our group since 1999.

In some cases (odd strength, odd cardinality, other conditions) this implies nonexistence results in the first open cases and in certain asymptotic process. Other applications lead to bounds (upper and lower) on the smallest and largest...
inner products. Still another consideration gives bounds on the covering radius of spherical designs.

**Better than average – using random points on the sphere for benchmarks**

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Random points on the sphere provide a useful benchmark for the quality of numerical integration and approximation methods based on “well-chosen” node configurations. A highly desired property would be to beat the average emerging form using random points.

Topics of the presentation will be numerical integration on the sphere using QMC designs ([3,5,6]), the quality of constructible “well-distributed” node sets ([1,2]), average energy and covering and separation properties of random points on the sphere ([4]).


Zeros and rate of convergence of interpolation rational approximants
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The classical Jentzsch-Szegő theorem on zeros of Taylor polynomials has been extended, among other situations, to row sequences of Padé approximants [4], rational functions of best uniform approximation [2], and Padé approximants with unbounded number of poles [1]. In all cases the zero limit distribution of the approximants turns out to be the equilibrium measure of a certain set.

The authors extend the Jentzsch-Szegő theorem to the case of a function \( f \) interpolated by multi-point Padé approximants. The interpolation process is carried out along an arbitrary table of points lying on a compact set on a neighborhood of which \( f \) is analytic. In this case the limit distribution of the zeros is no longer an equilibrium measure but it is given by a generalized balayage measure depending on the interpolation points and the region of analyticity of the function \( f \).

This result constitutes an extension to Padé approximants of a previous work [3] dealing with interpolating polynomials.

Moreover, should there exist a subsequence of approximants for which the rate of convergence is faster on a given continuum that does not reduce to a single point, they prove that such a subsequence is overconvergent.


Alternative Jacobi polynomials and their counterparts
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We describe classical properties of alternative Jacobi polynomials; consider some of their subsets of particular interest, as well as their exponential and rational analogs on half lines.
The Narrow Escape Potential, Optimal Arrangements of \( N \) Traps on the Unit Sphere, and the Dilute Trap Limit for \( N \gg 1 \).

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A narrow escape problem is considered to calculate the mean first passage time needed for a Brownian particle to leave a domain through one of its \( N \gg 1 \) small boundary windows (traps). Such problems arise in chemical and cell-biological modeling. The MFPT satisfies a strongly heterogeneous Dirichlet-Neumann boundary value problem for the Poisson equation.

For the spherical domain, putative optimal arrangements of \( N \gg 1 \) equal small boundary traps that minimize the asymptotic MFPT are computed. An explicit expression (pairwise interaction energy) that needs to be minimized follows from the asymptotic theory; it is presented and discussed. Examples that use traps of different sizes are also considered.

A dilute trap limit of homogenization theory when \( N \to \infty \) is used to approximate the strongly heterogeneous boundary value problem with a spherically symmetric Robin problem. For the latter, the exact solution is readily found. Parameters of the Robin homogenization problem are computed that capture the first four terms of the asymptotic MFPT expression. Close agreement of asymptotic and homogenization MFPT values is demonstrated. The homogenization approach provides a radically faster way to estimate the MFPT, since it is given by a simple formula, and does not involve computationally expensive global optimization to determine actual locations of the \( N \gg 1 \) boundary traps.

Minimax principle and lower bounds in \( H^2 \) rational approximation

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I will present methods allowing one to derive some lower bounds in rational approximation of given degree to functions in the Hardy space \( H^2 \) of the disk. The algorithms that solve the problem of best rational approximation to a function usually only find a ’candidate’, in the sense that they find a local minimum of the criterion, with good hope that this minimum is indeed global. Providing a good lower bound to this problem is thus an interesting complement to such solvers, as it gives an interval of confidence for the true optimal value.

A first method is based on Adamjan-Arov-Krein theory and leads to a lower bound that is fairly easy to compute from a numerical point of view. This bound is also of theoretical interest, as it allowed us to derive asymptotic errors rates in approximation to Blaschke products and to Cauchy integrals on geodesic arcs.
A second method is based on linearized errors and leads to more involved numerical computations, less easily implemented. Yet, results on a few examples show that this method can compute fairly sharp lower bounds, with respect to the true optimal error.

I will present both methods and discuss the difficulties in their practical implementation. They will be illustrated by numerical results on a few examples.

**Sums of monomials with large Mahler measure**

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For \( n \geq 1 \) let \( A_n \) be the collection of all sums of \( n \) distinct monomials. These polynomials are also called Newman polynomials. If \( \alpha < \beta \) are real numbers then the Mahler measure \( M_0(Q, [\alpha, \beta]) \) is defined for bounded measurable functions \( Q \) on \( [\alpha, \beta] \) as

\[
M_0(Q, [\alpha, \beta]) := \exp\left(\frac{1}{\pi \alpha} \int_{\alpha}^{\beta} \log |Q(e^{it})| \, dt \right).
\]

Let \( I := [\alpha, \beta] \).

In this talk, we examine the quantities

\[
L_n^0(I) := \sup_{P \in A_n} \left( \frac{M_0(P, I)}{\sqrt{n}} \right) \quad \text{and} \quad L_0^0(I) := \lim \inf_{n \to \infty} L_n^0(I)
\]

with \( 0 < |I| := \beta - \alpha \leq 2\pi \).

**Vector-valued inequalities for operators associated to Jacobi and Laguerre functions**

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The aim of this talk is to show vector-valued extensions of operators related to Fourier series of certain orthonormal systems and give some applications. We deal with fractional integrals for Fourier-Jacobi and Fourier-Laguerre expansions, and the Riesz transform for Fourier-Laguerre series.

The key point is to obtain bounds for the kernels of the operators which are \textbf{explicit} in the type parameters of Jacobi or Laguerre functions.
Uniformization of the isospectral torus for finite-gap Jacobi matrices
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A constructive function theoretic approach to the inverse problem for finite-gap Jacobi matrices is presented. The so-called Schottky-Klein prime function is used to give a uniformization of the hyperelliptic curve associated with the isospectral torus of finite-gap Jacobi matrices. Explicit formulas for the inverse spectral problem are derived. Computational aspects will also be discussed.

Exponential analysis, Sparse interpolation and Padé approximation
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Many real-time experiments involve the measurement of signals which fall exponentially with time. The approximation problem is then to determine, from such measurements, the number of terms $n$ and the value of all the parameters in the exponentially damped model

$$\phi(t) = \sum_{i=1}^{n} \alpha_i \exp(\phi_i t), \quad \alpha_i = \beta_i + i \gamma_i, \quad \phi_i = \psi_i + i \omega_i.$$  

Here $\psi_i, \omega_i, \beta_i$ and $\gamma_i$ are respectively called the damping, frequency, amplitude and phase of each exponential term.

The problem of multi-exponential modelling is an inverse problem and may be ill-posed. Our purpose is to propose a regularization of this ill-posed problem, making use of tools such as the singular value decomposition and various convergence results for Padé approximants. By means of the latter we can make the algorithm very robust [3]. By doing so we also extend the applicability of what is commonly known in the applied sciences as the Padé-Laplace method [1].

The analysis of band-limited signals has given rise to a well-developed theory of resolution, associated with the names of Shannon and Nyquist [4]. It states that, if $\Omega/2$ is the highest frequency present in its spectrum, the frequency content of a signal is completely determined by its values at equidistantly spaced time points, $2\pi/\Omega$ apart. A coarser time grid causes aliasing, identifying higher frequencies with lower frequencies without being able to distinguish between them. As a consequence, the exponential analysis as it is used today, suffers a similar frequency resolution limitation.

In addition, in the presence of noise, different exponential decays $\psi_i$ cannot be resolved if the ratio of the damping factors is less than some threshold [2], in other words when the damping constants are too much alike. It is generally
believed that none of the above limitations can be overcome. However, we offer a technique that overcomes both limitations, while at the same time, it improves the conditioning of the numerical problems involved. The technique is exploiting aliasing rather than avoiding it and breaks the Shannon-Nyquist sampling rate that stands since [4]! It is the result of combining ideas from exponential analysis and Padé approximation with results from sparse interpolation in computer algebra.

References

Critical edge behavior and the Bessel to Airy transition in the singularly perturbed Laguerre unitary ensemble
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In this paper, we study the singularly perturbed Laguerre unitary ensemble

\[ \frac{1}{Z_n} \left| \text{det } M \right|^\alpha e^{-\text{tr} V_t(M)} dM \]

with \( V_t(x) = x + t/x, \ x \in (0, +\infty) \) and \( t > 0 \). Due to the effect of \( t/x \) for varying \( t \), the eigenvalue correlation kernel has a new limit instead of the usual Bessel kernel at the hard edge 0. This limiting kernel involves \( \psi \)-functions associated with a special solution to a new third-order nonlinear differential equation. The transition of this limiting kernel to the Bessel and Airy kernels is also studied when the parameter \( t \) changes in a finite interval \( (0, d] \). Our approach is based on Deift-Zhou nonlinear steepest descent method for Riemann-Hilbert problems.
Large degree asymptotics of polynomials orthogonal with respect to oscillatory weights
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We present recent results on the asymptotic behavior and asymptotic zero distribution (as the degree tends to infinity) of polynomials $p_n(x)$ that are orthogonal with respect to a weight function $w(x)$ that is oscillatory on the real axis. The two main examples will be a complex exponential weight on $[-1,1]$, with a potentially large oscillatory frequency, and Bessel functions of order $\nu \geq 0$ on $[0,\infty)$. The tools used for the analysis are logarithmic potential theory in the complex plane and the $S$-property, together with the Riemann-Hilbert formulation and the Deift-Zhou steepest descent method.

Sharp Nikol’skii inequality for algebraic polynomials on an interval and on a sphere
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Let $P_n$ be the set of algebraic polynomials of degree $n \geq 1$. On this set, consider the uniform norm $\|f\|_C = \max\{|f(t)| : t \in [-1,1]\}$ and the integral $L_\alpha^q$-norm with ultraspherical weight $\|f\|_{L_\alpha^q} = (\int_{-1}^1 |f(t)|^q (1-t^2)^{\alpha}dt)^{1/q}$. On the set $P_n$, we study the sharp Nikol’skii inequality

$$\|f\|_C \leq M_n(q,\alpha)\|f\|_{L_\alpha^q}. \tag{1}$$

Let $g_n$ be the polynomial in one variable with unit leading coefficient that deviates least from zero in the space $L_\alpha^q$ of functions $f$ such that $|f|^q$ is summable over $(-1,1)$ with the Jacobi weight $\psi(t) = (1-t)^{\alpha + 1}(1+t)^\alpha$.

**Theorem.** For $n \geq 1, 1 \leq q < \infty, \text{ and } \alpha > 0$, the polynomial $g_n$ is extremal in inequality (1).

Inequality (1) is related to the Nikol’skii inequality

$$\|f\|_{C(S^{m-1})} \leq C_n(q,m)\|f\|_{L_q(S^{m-1})} \tag{2}$$

on the set $P_{n,m}$ of algebraic polynomials in $m$ variables of degree $n$ between the uniform norm and the $L_q$-norm of polynomials on the unit sphere $S^{m-1}$ of the Euclidean space $\mathbb{R}^m$. More precisely [1], for $\alpha = (m-3)/2, m \geq 3$, the
best constants in these inequalities are connected by the relation \( M_n(q, \alpha) = |S^{m-2}|^{1/q}C_n(q, m) \), where \( |S^{m-2}| \) is the classical measure of the sphere. Moreover, in this case, the polynomial \( \varrho_n \) is the unique extremal polynomial in inequality (1) and, as a zonal polynomial, is (in a certain sense) unique extremal in inequality (2).

The results were obtained by the author jointly with V.V.Arestov.

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The estimates on the entropy for the spectral measure of the multidimensional Schrödinger operator.

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We will consider the multidimensional Schrödinger operator and study the lower estimates for the entropy integral of the spectral measure for compactly supported \( L^2 \) function.

Zero distribution of polynomials satisfying a differential-difference equation

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In this talk, we investigate the asymptotic distribution of the zeros of polynomials \( P_n(x) \) satisfying a first order differential-difference equation. We give several examples of orthogonal and non-orthogonal families.
External Field Problems on the Hypersphere and Optimal Point Separation
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Optimal spherical configurations have wide-ranging application in science. In this talk we shall briefly survey various applications and focus on minimal energy configurations and in particular on their separation properties. Our techniques naturally lead to investigation of optimal configurations in the presence of external field. The separation properties of such configurations will also be established.

Common and interlacing zeros of families of Laguerre polynomials
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We discuss the common zeros of the Laguerre polynomials $L_n^{(\alpha)}$ and $L_{n-k}^{(\alpha+t)}$, considering them as functions of $t$. These common zeros are useful in discussing the interlacing of the zeros. Our main result is that if $\alpha \geq 0$, and $k$ is a positive integer with $1 \leq k \leq n-2$, then for each $t$ in the interval $0 \leq t \leq 2k$, excluding the values of $t$ for which $L_n^{(\alpha)}$ and $L_{n-k}^{(\alpha+t)}$ have a common zero, the zeros of these two polynomials interlace. Moreover, the interval $0 \leq t \leq 2k$ is the largest possible interval in which this interlacing property holds for all $n$. We use the interlacing concept in an extended sense, originally due to Stieltjes.

Extending Askey tableau by the inclusion of Krall and exceptional polynomials.
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Krall and exceptional polynomials are two of the more important extensions of the classical families of Hermite, Laguerre and Jacobi. On the one hand, Krall or bispectral polynomials are orthogonal polynomials which are also eigenfunctions of a differential operator of order bigger than two (and polynomial coefficients). The first examples were introduced by H. Krall in 1940, and since the eighties a lot of effort has been devoted to this issue (with contributions by L.L. Littlejohn, A.M. Krall, J. and R. Koekoek, A. Grunbaum and L. Haine (and collaborators), K.H. Kwon (and collaborators), A. Zhedanov, P. Iliev, and many
others. On the other hand, exceptional polynomials are orthogonal polynomials which are also eigenfunctions of a second order differential operator, but they differ from the classical polynomials in that their degree sequence contains a finite number of gaps, and hence the differential operator can have rational coefficients. In mathematical physics, these functions allow to write exact solutions to rational extensions of classical quantum potentials. Exceptional polynomials appeared some five years ago, but there has been a remarkable activity around them mainly by theoretical physicists (with contributions by D. Gomez-Ullate, N. Kamran and R. Milson, Y. Grandati, C. Quesne, S. Odake and R. Sasaki, and many others). Taking into account these definitions, it is scarcely surprising that no connection has been found between Krall and exceptional polynomials. However, if one considers difference operators instead of differential ones (that is, the discrete level of Askey tableau), something very exciting happens: Duality (i.e., swapping the variable with the index) interchanges Krall discrete and exceptional discrete polynomials. This unexpected connection of Krall discrete and exceptional polynomials allows a nice and important extension of Askey tableau. Also, this worthy fact can be used to solve some of the most interesting questions concerning exceptional polynomials; for instance, to find necessary and sufficient conditions such that the associated second order differential operators do not have any singularity in their domain.

Weak limits and ratio asymptotics for weighted sums of orthogonal polynomials related to spherical harmonics
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For orthogonal polynomials on the real line and on the unit circle, the study of ratio asymptotics and weak limits has a long and fruitful history (see [1] and the references therein). The aim of this presentation is to consider such limit results for weighted sums of polynomials orthogonal with respect to a varying weight function. The properties of the orthogonality measures and the coefficients in the summation are chosen in such a way that the weight function of the ultraspherical polynomials arises in the corresponding asymptotic limit. For the proof of these asymptotic limits we use the powerful machinery of spectral theory for orthogonal polynomials (as given in [1]) on the one hand and particular addition theorems for Jacobi and Gegenbauer polynomials on the other. As an application we show how such weak limits provide information on the approximability of functions on the sphere with space-frequency localized basis functions.

Convergent interpolatory quadrature schemes
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A connection between interpolatory quadrature formulae and fourier series of orthogonal polynomials is found. Such result is used to find a wide class of convergent schemes of quadrature rules.

Some energy integrals with external fields related to heights
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We will discuss the minimization of a certain energy integral with external fields which is related to the height of algebraic numbers. The solution of this problem (over all local fields, but particularly, over the real line) allows us to give new lower bounds for the height of totally real and p-adic numbers.

Continuum Schrödinger operators generated by aperiodic subshifts
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We will discuss continuum analogues of substitution Hamiltonians – specifically, we will discuss Schrödinger operators on the real line whose potentials are described by an ergodic subshift over a finite alphabet and a rule that replaces symbols of the alphabet by compactly supported potential pieces. In this setting, the spectrum and the spectral type are almost surely constant, and one can identify the almost sure absolutely continuous spectrum with the Lebesgue essential closure of the set of energies with vanishing Lyapunov exponent. Using this and results of Damanik-Lenz and Klassert-Lenz-Stollmann, we can show that the spectrum is a Cantor set of zero Lebesgue measure if the subshift satisfies a precise combinatorial condition due to Boshernitzan. We will discuss the specific case of operators over the Fibonacci subshift in detail.
Coherence relations for orthogonal polynomials on the unit circle
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We present some results regarding coherent pairs of linear functionals on the unit circle. In particular, we present a partial classification of the measures associated to such coherent pairs for some particular cases. Some algebraic properties are derived.

Polynomials with no zeros on a face of the bidisk.
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We present a Hilbert space geometric approach to the problem of characterizing the positive bivariate trigonometric polynomials that can be represented as the square of a two variable polynomial possessing a certain stability requirement, namely no zeros on a face of the bidisk. The characterization is in terms of a certain orthogonal decomposition the Hilbert space must possess called the “split-shift orthogonality condition.”

Discretization of stochastic heat equations with multiplicative noise on the unit sphere.
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In this work, we investigate the discretization of a class of stochastic heat equations on the unit sphere with multiplicative noises. A spectral method is used for spatial discretization while an implicit Euler scheme is used for time discretization. Some numerical experiments will be given. This is an on-going work with Christoph Schwab and Ian Sloan.
Hardy–Littlewood type inequalities for fractional integrals

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Let $L_{p}^{(a,b)}$, $1 \leq p < \infty$, be the space of all functions $f$ measurable on $[-1; 1]$ with the finite norm

$$\|f\|_{L_{p}^{(a,b)}} = \left( \int_{-1}^{1} |f(x)|^{p} w^{(a,b)}(x) \, dx \right)^{1/p},$$

where $w^{(a,b)}(x) = (1-x)^{a}(1+x)^{b}$, $a \geq b \geq -1/2$. For $c \geq d \geq -1/2$, denote by $\{\psi_{n}^{(c,d)}\}$ the system of Jacobi polynomials orthonormal on $[-1; 1]$ with respect to the weight $w^{(c,d)}$.

With a function $f \in L_{1}^{(c,d)}$, we associate the Fourier–Jacobi expansion

$$f(x) \sim \sum_{n=0}^{\infty} \hat{f}_{n}\psi_{n}^{(c,d)}(x), \quad \hat{f}_{n} = \int_{-1}^{1} f(x)\psi_{n}^{(c,d)}(x) w^{(c,d)}(x) \, dx.$$  

The fractional integral of order $s > 0$ of the function $f$ is a function $g = I_{s}^{(c,d)} f \in L_{1}^{(c,d)}$ such that

$$g(x) \sim \hat{f}_{0} + \sum_{n=1}^{\infty} (n(n+c+d+1))^{-s/2} \hat{f}_{n}\psi_{n}^{(c,d)}(x).$$

Muckenhoupt [3] investigated the properties of $I_{s}^{(c,d)}$ as an operator from $L_{p}$ into $L_{q}$ with some weights and $s = 1/p - 1/q$, $1 < p < q < \infty$, i.e. when the order $s$ is independent of weights. Askey and Wainger [1], and Bavinck [2] found that $I_{s}^{(c,d)}$ is a bounded operator from $L_{p}^{(a,b)}$ into $L_{q}^{(a,b)}$ for $(c,d) = (a,b)$ and $s = (2a+2)(1/p - 1/q)$. We are interested in the case of $(c,d) \neq (a,b)$ and prove that under some conditions on $(c,d)$, $(a,b)$ and $s = (2a+2)(1/p - 1/q)$ the operator $I_{s}^{(c,d)}$ is a bounded operator from $L_{p}^{(a,b)}$ into $L_{q}^{(a,b)}$.

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Lower bounds for spherical designs
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Spherical $t$-designs are defined as equal weight quadrature rules, which yield exact integration for all polynomials of degree $\leq t$ on the sphere $S^d$. Properties of designs have attracted new interest by the recent proof of the existence of designs with $O(t^d)$ number of points by Bondarenko, Radchenko, and Viazovska. We will report on corresponding lower bounds for the number of points needed and point out relations to results on packing densities in $\mathbb{R}^d$.

Lagrange Polynomials, Reproducing Kernels and Cubature in Two Dimensions
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We obtain by elementary methods conditions for a $k$-dimensional cubature formula to hold exactly for all polynomials of degree up to $2m-1$ when the nodes of the formula have Lagrange polynomials of degree at most $m$. The conditions are that any polynomial of degree $m$ that vanishes on all of the nodes is an orthogonal polynomial of degree $m$ and that the Lagrange polynomial at each node is a scalar multiple of the reproducing kernel of degree $m-1$ evaluated at the node plus an orthogonal polynomial of degree $m$. Stronger conditions are given for the case where the cubature formula holds exactly for all polynomials of degree up to $2m$.

This result applies in one dimension to obtain a quadrature formula where the nodes are the roots of a quasi-orthogonal polynomial of order 2. We give 16 examples of specific quadrature formulas derived from the Chebyshev polynomials of kinds 1-4 where the nodes and Christoffel numbers can be obtained as simple expressions. An argument of Bojanov and Petrova applies to obtain a general bivariate cubature formula where the Christoffel numbers are twice the product of the Christoffel numbers for the one dimensional case. Our formula includes previous results for the Chebyshev and Geronimus nodes.
Distribution of Poles of Tritronquée Solutions to the First Painlevé Equation

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The first Painlevé equation admits a family of special solutions called tritronquée solutions, which play important roles in quantum mechanics, fluid mechanics, and plasma physics. Based on numerical evidence, Dubrovin, Grava, and Klein conjectured that the tritronquée solutions have no pole in the complex plane except in a sector of angle $2\pi/5$. In this presentation, we introduce a rigorous computational method for finding accurate global approximations of the tritronquée solutions with rigorous error bounds. This constructive approach allows us to prove the aforementioned conjecture, as well as part of Joshi and Kitaev’s conjectures on numerical values of the tritronquée solutions. The general method has been applied to spectral problems arising in the theory of nonlinear equations, as well as boundary layer problems in fluid mechanics. We will also discuss possible applications to the study of homoclinic orbits and stabilities and singularities of PDEs. This presentation is based on the joint work with Prof. Ovidiu Costin and Prof. Saleh Tanveer.

Thin Plate Spline Interpolation on the Unit interval

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It is known that the thin plate spline (TPS) interpolant to a sufficiently smooth function on the scaled infinite grid $h\mathbb{Z}$ provides $O(h^3)$ accuracy. However, when the function is sampled on a finite grid (at equally spaced points on the unit interval) then numerical evidence indicates that the presence of a boundary causes a deterioration in the accuracy from $O(h^3)$ to $O(h^{3/2})$. In this talk we will report results from further numerical experiments of this situation. For instance, we will show that specific target functions can be constructed for which the TPS interpolant converges at the faster rate of $O(h^{5/2})$. Furthermore, we propose an alternative TPS interpolation method which succeeds in achieving $O(h^{5/2})$ accuracy for elementary target functions. The theoretical explanations of these intriguing numerical results are open issues that we are currently investigating.
Constructing bispectral orthogonal polynomials from the classical discrete families of Charlier, Meixner and Krawtchouk
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Given a sequence of polynomials \((p_n)_n\), an algebra of operators \(A\) acting in the linear space of polynomials and an operator \(D_p \in A\) with \(D_p(p_n) = np_n\), we form a new sequence of polynomials \((q_n)_n\) by considering a linear combination of \(m + 1\) consecutive \(p_n\): \(q_n = p_n + \sum_{j=1}^{m} \beta_{n,j} p_{n-j}\). Using the concept of \(D\)-operator, we determine the structure of the sequences \(\beta_{n,j}, j = 1, \ldots, m\), in order that the polynomials \((q_n)_n\) are eigenfunctions of an operator in the algebra \(A\). As an application, from the classical discrete families of Charlier, Meixner and Krawtchouk we construct orthogonal polynomials \((q_n)_n\) which are also eigenfunctions of higher order difference operators.

A hypergeometric basis for the Alpert multiresolution analysis
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I will describe an explicit orthonormal basis of piecewise \(i+1F_i\) hypergeometric polynomials for the Alpert multiresolution analysis. The Fourier transform of each basis function can be written in terms of \(2F_3\) hypergeometric functions. Moreover, the entries in the matrix equation connecting the wavelets with the scaling functions are balanced \(4F_3\) hypergeometric functions evaluated at 1, which allows to compute them recursively via three-term recurrence relations.

The above results lead to a variety of new interesting identities and orthogonality relations reminiscent to classical identities of higher-order hypergeometric functions and orthogonality relations of Wigner 6j-symbols.

The talk will based on joint work with Jeff Geronimo.
Extrapolation inequalities in Lebesgue type spaces
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A weight $w$ is said to belong to the class $B^*_p$ if the inequality
$$\int_0^r \left( \frac{t}{x} \right)^p w(x) \, dx \leq c \int_0^r w(x) \, dx$$
holds for all $r > 0$ and for some $c > 0$. It is known that $B^*_p$ weights characterize the $L^p$-boundedness of the conjugate Hardy averaging operator
$$A^* f(x) = \int_x^\infty \frac{f(t)}{t} \, dt$$
for non-increasing functions, i.e., the inequality
$$\|A^* f\|_{L^p_w} \leq c \|f\|_{L^p_w}, \quad f \downarrow, \ 1 < p < \infty$$
holds. Also it is known that if the inequality (1) holds for some $p \in (1, \infty)$, then it holds for all $p \in (1, \infty)$ giving the extrapolation effect. We shall discuss these results and similar recent results proved for more general operator
$$S^*_\phi f(x) = \int_x^\infty f(t) \frac{\phi(t)}{\Psi(t)} \, dt,$$
where $\phi(x) = \int_0^x \phi(t) \, dt$ and $\phi$ has certain suitable conditions. Moreover, we shall be discussing these results in the context of grand Lebesgue spaces $L^{p^*}$ as well.

On the Asymptotics of Discrete Minimal Riesz Energy on Regular Compact Sets
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Discrete minimal $s$-Riesz energy ($s \geq 0$) of a system of $n$ points on a compact set $\Omega \subseteq \mathbb{R}^d$ has been intensively studied during the last years. For those cases in which its continuous counterpart exists (i.e., $0 \leq s < d - 1$) the normalized discrete minimal $s$-Riesz energy is known to converge to the continuous $s$-Riesz energy as $n$ tends to infinity. For the $(d - 1)$-dimensional sphere in $\mathbb{R}^d$ sharp error bounds are known.

In this talk we discuss techniques for a possible extension of these error bounds to more general compact sets $\Omega$. To this end different regularity assumptions are discussed and some partial results are presented. Particularly for differentiable manifolds, the use of tubular neighborhoods and a continuity statement for the surface measure of intersections between certain balls and the manifold are considered as devices for deriving suitable error bounds.

Finally numerical results are presented in which extremal points on certain smooth and non-smooth manifolds are computed by an interior point method. In addition to the minimal discrete energy the structure of the set of local
Quasi-orthogonality and real zeros of some $2F_2$ and $3F_2$ polynomials

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We prove the quasi-orthogonality (cf. [1]) of some general classes of hypergeometric polynomials of the form

$$pF_q\left(-n, \beta_1 + k, \alpha_3, \ldots, \alpha_p \mid \beta_1, \ldots, \beta_q \right) = \sum_{m=0}^{n} \frac{(-n)_m (\beta_1 + k)_m (\alpha_3)_m \ldots (\alpha_p)_m x^m}{(\beta_1)_m \ldots (\beta_q)_m m!}$$

for $k \in \{1, 2, \ldots, n - 1\}$ which do not appear in the Askey scheme for hypergeometric orthogonal polynomials (cf. [3]). Our results include, as special cases, the polynomials

$$f_n(a, x) = 3F_2\left(-n, n + 1, a \mid \frac{1}{2}, 1 \right) = \sum_{m=0}^{n} \frac{(-n)_m (n + 1)_m (a)_m x^m}{(\frac{1}{2})_m (1)_m m!}$$

with $a = 2$ and $a = 3/2$ considered by Dickinson [2] in 1961. Our method makes use of the orthogonality of lower order $p-1F_{q-1}$ polynomials and an important characterisation of quasi-orthogonal polynomials as a linear combination of orthogonal polynomials. Results on the location and interlacing of the real zeros related to the quasi-orthogonality of the polynomials under consideration are also discussed.

On a subclass of harmonic univalent functions defined by convolution
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In the present talk, we introduce and study a subclass of harmonic univalent functions defined by convolution and integral convolution. Coefficient bounds, extreme points, distortion bounds, convolution conditions and convex combinations are determined for functions in this family.

On asymptotically sharp Markov inequality on general compact sets
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Coauthors: Bela Nagy and Vilmos Totik

Markov type inequality for polynomials is widely used in approximation theory. Its asymptotically sharp form for unions of finitely many intervals has been found in 2001. In this talk we discuss this asymptotic form on arbitrary compact subsets of the real line satisfying an interval condition. This sharp Markov factor is formulated in terms of the density of the equilibrium measure.

We also present a sharp local version of Schur’s inequality.
This is based on a joint work with Vilmos Totik and Bela Nagy.
The work was supported by the European Research Council Advanced grant No. 267055.

N point configurations on the sphere with (putatively) optimal Riesz energy: A concavity study
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A partly rigorous, yet mostly empirical study of the second discrete derivative of the map from the positive integers (bigger than 1) into the (putatively) minimal average (standardized) Riesz pair energy is presented. The following results are reported: (a) the minimal average Riesz pair energy is locally strictly concave if the Riesz parameter $s=-2$ (corresponding to repulsive harmonic pair interactions); (b) empirically it seems to be locally strictly concave also for $s=-1$; (c) empirically and quasi-rigorously, local strict concavity fails for $s=0$, $s=1$, $s=2$, and $s=3$; (d) the set of convexity defects seems to be expanding with $s$; (e) based on the working hypothesis that (d) holds, a search for non-optimal Riesz
energy configurations in lists of computer generated putative minimizers found three whose energy was, in fact, non-optimal. A list of substantiated conjectures is presented, as well as a list of questions whose answer may lead to rigorous tests of optimality in computer-generated lists of putative optimizers.

Random matrices and multiple orthogonal polynomials
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Multiple orthogonal polynomials are polynomials that satisfy orthogonality conditions with respect to a number of measures. They arise in problems in analytic number theory, rational approximation, and more recently in the theory of random matrices. Random matrix ensembles that were studied from this perspective include random matrices with external source, the two matrix model, and the normal matrix model.

I will give an overview of this and then focus on a new example that involves products of random matrices with independent Gaussian distributed entries. This is conceptually the simplest example. The squared singular values of these matrices are described by multiple orthogonal polynomials with weights that are Meijer G-functions. The limiting behavior as the size of the matrices tends to infinity gives rise to new phenomena near the origin. This new work is joint with Lun Zhang.

Implications of Marden’s Theorem for Inscribed Ellipses
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We give a constructive proof for the existence of inscribed family of ellipses in convex n-gons for $3 \leq n \leq 5$ using Marden’s theorem. In the case of a pentagon, we show how Marden’s theorem can be made to exhibit simultaneously two ellipses, one inscribed in the pentagon and the other inscribed in its diagonal pentagon. The two ellipses are as intrinsically linked as are the pentagon and its diagonal pentagon. Our method uses the theory dual of curves.
On row sequences of Padé and Hermite-Padé approximation
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Let $f$ represent a formal Taylor series and let $D_m = \{z : |z| < R_m\}$ denote the largest disk centered at the origin to which $f$ can be extended as a meromorphic function with at most $m$ poles (if $f$ is divergent then $D_m = \emptyset$). A classical result due to R. de Montessus de Ballore states that if $D_m$ contains exactly $m$ poles then the $m$-th row of the Padé table converges to $f$ uniformly in the spherical metric on each compact subset of $D_m$. Moreover, the poles of that sequence of approximants converge to the poles of $f$ with geometric rate (taking account of the orders). A.A. Gonchar proved the converse statement. Another inverse type result had been established by E. Fabry many years ago. He proved that if the poles of the first row of the Padé table converge to some point $a \neq 0$ (without any assumption on the rate of convergence) then $R_0 = |a|$ and $a$ is a singular point of $f$. This was extended by S.P. Suetin to the $m$-th row. An analogue of Montessus de Ballore’s theorem was given by E.B. Saff for Hermite-Padé approximation. Motivated by it, we have been studying inverse type theorems for row sequences of Hermite-Padé approximants. In this talk we will review our findings in this direction.

Prony’s method in higher dimensions
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In 1795, Gaspard Riche de Prony presented a method for interpolating a sum of exponential functions. Closely related to Padé approximation, Prony’s method has found applications in the shape from moments problem, spectral analysis, and lately sparse sampling of digital signals with finite rate of innovation.

On the one hand, the modern least squares approach of exponential modeling has evolved quite significantly from Prony’s original version. On the other hand, the interesting connection between Prony’s method and error-correcting codes has led to the development of symbolic-numeric sparse polynomial interpolation, which has already exploited a generalized eigenvalue reformulation and a link to Rutishauser’s qd-algorithm. This connection further enables a generalization of variances of Prony to other basis functions.

Recall that a meromorphic function is a function analytic everywhere except at a set of isolated points that are called the poles of the function. Rutishauser’s qd-algorithm can determine the poles of a meromorphic function from its Taylor expansion. In the multivariate case such poles form a set of solutions of the
associated multivariate polynomial equations. The interdependence between the Taylor expansion and poles becomes less obvious because there can be various ways to order the multivariate Taylor coefficients.

Recent progress in the multivariate qd-algorithm expands our understanding in associating the convergence of multivariate poles to the different orderings of Taylor coefficients, among which a special case has been implemented in developing numerical multivariate polynomial factorization. Resorting to the link from qd to Padé leads us to a multivariate Prony’s method, which intricately involves higher-order tensors and their decompositions.

Discrepancy, separation and Riesz energy of finite point sets on compact connected Riemannian manifolds
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On a smooth compact connected $d$-dimensional Riemannian manifold $M$, if $0 < s < d$ then an asymptotically equidistributed sequence of finite subsets of $M$ that is also well-separated yields a sequence of Riesz $s$-energies that converges to the energy double integral, with a rate of convergence depending on the geodesic ball discrepancy. This generalizes a known result for the sphere.

On Finite Blaschke Products Interpolating on the Unit Circle
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Given $n$ distinct points $z_j$, $j = 1, 2, ..., n$, on the unit circle in the complex plane and given $n$ values $w_j$, $j = 1, 2, ..., n$, also on the unit circle, it is known that there exist finite Blaschke products $B_n$ with at most $n$ poles such that

$$B_n(z_j) = w_j, \quad j = 1, 2, ..., n.$$ 

There are cases where multiple solutions exist and, due to its nonlinear nature, the poles of all possible solutions are hard to characterize. C. Glader used Nevanlinna parametrization to describe all solutions. We will show a simpler set of parameters that may be employed to give a complete parametrization of the set of all the solutions. This talk will be based on joint research with R.N. Mohapatra and R. Puwakgolle.
Asymptotic Approximations of the Wilson Polynomials via Recurrence Relations
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In this talk, we discuss various asymptotic formulas for the Wilson polynomials as the degree grows to infinity. Our approach is based on the recurrence relation that the Wilson polynomials satisfy, and these approximations include the fixed-\(x\) asymptotics and uniform asymptotics around turning points for the polynomials, as well as the asymptotics of zeros. In particular, a Bessel type approximation is derived for the turning point at zero, and an Airy type approximation for the other turning point. If time is allowed, we shall also discuss the asymptotic approximations for the continuous Hahn and the continuous dual Hahn polynomials.

Uniform asymptotics of the Szegő-Askey polynomials
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In this talk, we would talk about the asymptotics of the Szegő-Askey polynomials as the degree \(n\) grows to infinity. Uniform asymptotic formulas in terms of special functions and elementary functions are obtained for \(z\) in three overlapping regions, which together cover the whole complex plane. Our method is based on a modified version of the Riemann-Hilbert approach introduced by Deift and Zhou.

Multiple orthogonal polynomials on a star
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Let \(p \geq 2\) be a fixed integer. In this talk I will describe asymptotic properties of a sequence of multiple orthogonal polynomials associated with a system of measures supported on star-like sets centered at the origin and consisting of \(p + 1\) equidistant rays. The system of measures has a Nikishin type structure.

A remarkable property of the sequence \(Q_n\) of polynomials that we consider (and the main motivation for its study) is the fact that it satisfies a three-term recurrence relation of order \(p + 1\) of the form \(xQ_n = Q_{n+1} + a_nQ_{n-p}\), for
certain positive coefficients $a_n$. Under certain assumptions on the structure of the system of orthogonality measures, we show that the sequence of recurrence coefficients $a_n$ has $p(p+1)$ periodic limits. This and related asymptotic results will be described.

**Probability Theory for Continued Fractions**
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Continued fractions show an amazing willingness to converge. Indeed, it is hard to find divergent continued fractions if we disregard the obvious ones – the periodic and limit periodic continued fractions of elliptic type. So an old dream of mine has been to prove that almost all continued fractions converge. Of course, a statement like that would obviously depend on how the phrase *almost all* is defined. For instance, a $\mu$-random continued fraction $K(a_n/b_n)$; i.e., its elements $(a_n, b_n)$ are picked randomly and independently from $\mathbb{C}^2$ according to some probability measure $\mu$, converges with probability 1 under mild conditions on the measure. This contributes somehow to the understanding of the nature of continued fractions with complex elements.

Another question is: what can we say about the probable values of such a random continued fraction? In other words, what can we say about the probability distribution of its limit? That is a much harder question, but results of this nature has applications also in a wider context.

In this talk we will look at some established results, some applications and some open questions.

**Approximation of sgn x by rational functions with fixed poles**
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Recently A.Eremenko and P. Yuditskii found explicit solutions of the best polynomial approximation problems of $\text{sgn}(x)$ over two intervals in terms of conformal mappings onto special comb domains. We give analogous solutions for the best approximation problems of $\text{sgn}(x)$ over two symmetric intervals by odd rational functions with fixed poles. Here the existence of the related conformal mapping is proved by using convexity of the comb domains along the imaginary axis.
Higher order Szegö theorems of arbitrary order
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We study the relation between a probability measure $\mu$ on the unit circle and its sequence of Verblunsky coefficients. Higher order Szegö theorems are equivalence statements relating decay conditions on the Verblunsky coefficients with integral conditions on the absolutely continuous part of the measure. They have immediate corollaries establishing presence of absolutely continuous spectrum under appropriate decay conditions. We will present a higher order Szegö theorem of arbitrary order; this is the first known result of this form in the regime of very slow decay, i.e. with $\ell^p$ conditions with arbitrarily large $p$.

A Naimark-type Theorem for Parseval Frame MRA Wavelets and Ideas Related to Shift-invariant Spaces
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A well known theorem of Naimark says that any Parseval frame on a Hilbert space can be realized as the orthogonal projection of an orthonormal basis in some larger Hilbert space. Recently, an analogous structural theorem was shown for Parseval frame MRA wavelets at the level of the associated scaling functions. In particular, the scaling function for any Parseval frame MRA wavelet can be realized as the orthogonal projection of the scaling function of a so-called maximal Parseval frame MRA. For a large subclass of Parseval frame MRAs, this says that the scaling function is the projection of the scaling function of an orthonormal MRA wavelet. In this talk, we will discuss some of the ideas and definitions surrounding the above statements. Time permitting, we will discuss an open problem from shift-invariant space theory related to Carleson’s theorem about convergence of Fourier series.
Numerical computation of orthogonal polynomials of equilibrium measures on Cantor sets, with application to conformal mappings and to the isospectral torus.

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I will present numerical techniques that permit to reliably compute orthogonal polynomials associated with the equilibrium measure, in logarithmic potential theory, living on finite gap and Cantor sets, the latter obtained as limits of the former. The asymptotic properties of these polynomials—that we calculate up to very large orders—become a means of obtaining Green’s functions and conformal mappings, that are of interest in constructive function theory: I will display pictures of these mappings that reveal interesting details. Finally, I will show that these techniques also allow to compute the isospectral torus of Jacobi matrices for finite gap sets and for Cantor sets in the family of Iterated Function Systems. These numerical tools have been developed as an aid to the solution of open problems concerning singular continuous measures and their Fourier transforms, which I will briefly describe.

This work is based upon, and extends the results in, the preprint Orthogonal polynomials of equilibrium measures supported on Cantor sets, arXiv:1311.4819 and the papers Computing the equilibrium measure of a system of intervals converging to a Cantor set, DRNA Vol. 6 (2013) 51 – 61, Direct and inverse computation of Jacobi matrices of infinite iterated function systems, Numerische Mathematik vol. 125 (2013) 705 – 731. Background material can be found in Fourier-Bessel functions of singular continuous measures and their many asymptotics, ETNA Vol. 25 (2006) 409–430.

Boundedness of \((C, 1)\) Means of the Generalized Series of the Second Kind for General Exponential Weights

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Let \(I\) be a finite or infinite interval containing 0, and let \(W\) be the weight function. Assume that \(W^2\) is a weight, so that we may define orthonormal polynomials corresponding to \(W^2\). Let \(s\) denote the \(m^{th}\) partial sums of the generalized series of the second kind. We investigate boundedness in \(L^p\) spaces of the \((C, 1)\) means of the Generalized Series of the Second Kind. The class of weights \(W^2\) considered includes even and non-even exponential weights.
On the symmetry property for the equilibrium measures
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It is well known that many asymptotic questions in the theory of orthogonal polynomials, random matrix models, or rational approximation of algebraic functions can be answered in terms of measures that satisfy some symmetry properties. Their support is connected with trajectories of certain quadratic differentials, and usually they can be characterized as a saddle point for suitable energy functionals on the complex plane. In many cases, establishing the existence of such measures is the key step in the solution of the original problem. From this perspective we discuss some aspects of this interplay between the energy extremality, symmetry and the global structure of trajectories.

Well conditioned and robust Padé approximants
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In a recent paper, Trefethen and al. [1] have proposed a method for computing a robust Padé approximant based on Singular Value Decomposition techniques. They observe numerically that these approximants are insensitive to perturbations in the data, and do not have so-called spurious poles, that is, poles with close-by zero or poles with small residues. A black box procedure for eliminating spurious poles would have a major impact on the convergence theory of Padé approximants since it is known that convergence in capacity plus absence of poles in some domain $D$ implies locally uniform convergence in $D$.

In this talk we will propose a mathematical analysis of these numerical phenomena. We will study forward and backward conditioning of the application going from the Taylor coefficients $(c_i)$ of the function to the vector of coefficients of the numerator and denominator of the Padé approximant and provide a proof for robustness. We will show that the conditioning of underlying rectangular Toeplitz and Sylvester like matrices plays an important role and and will prove the absence of spurious poles for the subclass of so-called well conditioned Padé approximants.

This is a joint work with B. Beckermann.

On the Fixed Point Theorem in $\mathbb{R}^2$
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We will discuss a possible function theory approach of solving a well-known topology problem.

Type I Hermite-Padé approximation for certain systems of meromorphic functions
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Sequences of type I Hermite-Padé approximants of certain systems of meromorphic functions are considered. These systems are made up of rational perturbations of Nikishin systems of functions. We obtain an extension of Markov’s convergence theorem for these cases.

Numerical differentiation and prediction of blood glucose levels in CGM.
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Coauthors: Valeriya Naumova and Sergei V. Pereverzyev

Let $f : [-1, 1] \to \mathbb{R}$ be continuously differentiable. We consider the question of approximating $f'(1)$ from given data of the form $(t_j, f(t_j))_{j=1}^M$ where the points $t_j$ are in the interval $[-1, 1]$. It is well known that the question is ill-posed, and there is very little literature on the subject known to us. We consider a summability operator using Legendre expansions, together with high order quadrature formulas based on the points $t_j$’s to achieve the approximation. We also estimate the effect of noise on our approximation. The error estimates, both with or without noise, improve upon those in the existing literature, and appear to be unimprovable. The results are applied to the problem of short term prediction of blood glucose concentration, yielding better results than other comparable methods.
Exceptional Hermite Polynomials
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Exceptional orthogonal polynomials (so named because they span a non-standart polynomial flag) are defined as polynomial eigenfunctions of Sturm-Liouville problems. By allowing for the possibility that the resulting sequence of polynomial degrees admits a number of gaps, we extend the classical families of Hermite, Laguerre and Jacobi. In recent years the role of the Darboux (or the factorization) transformation has been recognized as essential in the theory of orthogonal polynomials spanning a non-standard flag. In this talk we will focus on exceptional Hermite polynomials: their regularity properties, asymptotics of zeros and their relation to the recent conjecture that ALL exceptional orthogonal polynomials are related via factorization transformations to classical orthogonal polynomials.

On the leading coefficient of polynomials orthogonal over domains with corners.
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We consider polynomials orthogonal with respect to area measure over a domain bounded by a piecewise analytic Jordan curve. It was proven recently by N. Stylianopoulos that the leading coefficient $\gamma_n$ of the $n$th orthonormal polynomial satisfies the asymptotic formula

$$\frac{\gamma_n c^{n+1}}{\sqrt{\frac{n+1}{\pi}}} - 1 = O(1/n)$$

as $n \to \infty$, where $c$ is the logarithmic capacity for the boundary of the domain. We will show that the $O$-error term above is, in general, sharp by exhibiting a concrete example of a domain with corners for which there is a constant $M$ such that

$$\frac{\gamma_n c^{n+1}}{\sqrt{\frac{n+1}{\pi}}} - 1 \geq M(n+1)^{-1}, \quad n \geq 0.$$
Asymptotically sharp Bernstein type inequalities on different sets of the complex plane
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Coauthors: Sergei Kalmykov

Bernstein (or Riesz) type polynomial inequalities are well known. On the complex plane Bernstein inequality was extended to compact sets bounded by smooth Jordan curves and the asymptotically sharp constant can be expressed with the normal derivative of Green’s function. There is a general conjecture for Jordan arcs that the asymptotically sharp Bernstein factor can be expressed as the maximum of the two normal derivatives of Green’s function. It was proved for the subarcs, and later, general subsets of the unit circle. We also mention some partial results for analytic Jordan arcs and known inequalities for rational functions.

This is based on a joint work with Sergei Kalmykov.

This research was realized in the frames of TMOP 4.2.4. A/2-11-1-2012-0001 National Excellence Program Elaborating and operating an inland student and researcher personal support system. The project was subsidized by the European Union and co-financed by the European Social Fund.

Univariate weighted Leja sequences as building blocks for sparse multivariate interpolation
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We show that a type of weighted Leja sequence for unbounded domains with exponential weights in one dimension produces a sequence of interpolation nodes whose contracted empirical distribution converges to the weighted potential equilibrium measure. (We actually show a slightly stronger result: that the contracted sequence is asymptotically weighted Fekete.) In particular, this implies that the sequence has asymptotic distribution identical to the zeros for the polynomial family orthogonal under the weight squared.

We use this latter observation as motivation for use of the univariate sequence as a composite rule for a multidimensional Smolyak sparse grid construction. We furnish several numerical examples that illustrate that this method is competitive with, and in many cases superior to, more standard Smolyak sparse grid methods currently used in the applied community.
Part II. A Novel Galerkin Method for Solving PDEs on the Sphere
Using Highly Localized Kernel Bases
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The main goal of this paper is to introduce a novel meshless kernel Galerkin
method for numerically solving partial differential equations on the sphere.
Specifically, we will use this method to treat the partial differential equation for
stationary heat conduction on $S^2$, in an inhomogeneous, anisotropic medium.
The equation for this heat-flow is $Lu = -\text{div}(a \nabla u) + b(x)u$, where \text{div} and $\nabla$
are the divergence and gradient on $S^2$, and $a$ is a rank 2 positive definite tensor
on $S^2$. The Galerkin method used to do this employs spatially well-localized,
“small footprint,” robust bases for the associated kernel space. The stiffness
matrices arising in the problem have entries decaying exponentially fast away
from the diagonal. Discretization is achieved by replacing the stiffness matrix
with one whose entries are computed by a very efficient kernel quadrature for-
mula for the sphere. The discretized stiffness matrix retains the exponential
decay in its off diagonal entries.

Plancherel-Rotach formulas and zero asymptotics for characteristic
polynomials of random matrices
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The behavior of the eigenvalues of random matrices is a large subject of re-
search in random matrix theory and it is closely connected to the behavior of the
corresponding average characteristic polynomials and their zeros. We study the
asymptotics of polynomials associated with Wishart type products of complex
Gaussian random matrices. These polynomials form a general class of multi-
ple orthogonal hypergeometric polynomials generalizing the classical Laguerre
polynomials. The proofs are based on a complex multivariate steepest descent
analysis. After suitable rescaling the asymptotic zero distributions for the poly-
nomials are studied and shown to coincide with the Fuss-Catalan distributions.
Moreover, introducing appropriate coordinates, elementary and explicit repre-
sentations are obtained for the densities as well as for the distribution functions
of the Fuss-Catalan distributions of general order.
Application of Freud weights for solving option pricing problems
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We develop a new discretization method for pricing options under the Black-Scholes (BS) model. To this end, we approximate the partial differential equation in the space variable by an interpolation method based on Freud weights coupled with a fully implicit time-stepping scheme. The method is shown to be an alternative to other existing procedures for the numerical approximation for pricing options. Numerical results are presented to demonstrate the accuracy of our method.

Equilibrium problems in some rational external fields
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In this talk, equilibrium measures in the presence of rational external fields, i.e. whose derivatives are rational functions, are considered. Our main interest is studying the dynamics for the equilibrium measure and its support when the total mass of the measure or other parameters in the external field vary. We pay special attention to the situations where the number of cuts (intervals) comprising the support changes. This talk is related to a joint work with J. Sánchez Lara, from Granada University (Spain).

On Approximation Properties of Kantorovich-type Sampling Operators
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For bounded uniformly continuous functions $f \in C(\mathbb{R})$ the generalized sampling operators are given by $(t \in \mathbb{R}; W > 0)$

$$(S_W f)(t) := \sum_{k=-\infty}^{\infty} f(k/W) \varphi(Wt - k)$$
and for measurable functions $f \in L^p(\mathbb{R})$ ($1 \leq p \leq \infty$) the corresponding Kantorovich-type sampling operators are given by ($t \in \mathbb{R}; W > 0; n \in \mathbb{N}$)

$$(S_{W,n}^K f)(t) := \sum_{k=-\infty}^{\infty} \int_{-\infty}^{\infty} f(u)nW\chi(nW(t-u))du\varphi(Wt-k)$$

with $\varphi, \chi \in L^1(\mathbb{R}), \int_{-\infty}^{\infty} \chi(u)du = 1, \sum_{k \in \mathbb{Z}} \varphi(u-k) = 1 (u \in \mathbb{R})$. Unlike operators $S_W$ the Kantorovich-type sampling operators do not depend on a single value $f(k/W)$, but contain an average of $f$ on a small interval around $k/W$.

We will take a closer look on Kantorovich-type sampling operators with particular $\varphi$ and $\chi$ kernels. We will use some results we have for operators $S_W$ to prove analogous results for operators $S_{W,n}^K$. The estimates of order of approximation by Kantorovich-type sampling operators will be given via the modulus of smoothness.

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**Exponentially-improved generalized asymptotic expansions for singularly perturbed BVPs**

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Matched asymptotic expansions have been widely applied to problems arising in various fields of sciences and engineers. Though with great success, such conventional method also causes considerable issues when exponentially small terms are ignored. In this talk, we consider a nonlinear two-point BVP

$$\varepsilon y''(x) + a(x)y'(x) + f(x,y(x)) = 0,$$

subject to

$$y(\alpha) = A, \quad y(\beta) = B,$$

where $\alpha, \beta, A$ and $B$ are constants with $\alpha < \beta$, $\varepsilon \ll 1$, $a(x)$ is a continuous function in $[\alpha, \beta]$ and $f(x,y)$ is a smooth function. Here $a(x)$ may be one-signed, equal to zero, or has zeros in $[\alpha, \beta]$. This problems covers a number of examples (e.g. Carrier’s problem) that have been studied previously. We want to provide a uniform asymptotic expansions containing exponential small terms in both the outer and the inner regions. The results are proved rigorously.
Extension of inverses of Gamma functions to the upper half plane
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The motivation is a result of Uchiyama, stating that the inverse of Euler’s Gamma function (on a half line) can be extended to a Pick function. Similar results in more general settings are discussed.

The Gamma function either increases or decreases on intervals between two consecutive critical points. For intervals of increase, the inverse has a Pick function extension. Similar results hold for the intervals of decrease.

A class of entire functions, related to Barnes’ double Gamma function, is introduced. Inverses of functions from this class (restricted to certain intervals of the real line) have Pick function extensions.

Parameter choice strategies for least-squares approximation of noisy smooth functions on the sphere
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We consider a polynomial reconstruction of smooth functions from their noisy values at discrete nodes on the unit sphere by a variant of the regularized least-squares method of C.An et al. SIAM J Numer. Anal. 50 (2012), 1513–1534. As nodes we use the points of a positive-weight cubature formula that is exact for all spherical polynomials of degree up to $2M$, where $M$ is the degree of the reconstructing polynomial. We first obtain a reconstruction error bound in terms of the regularization parameter and the penalization parameters in the regularization operator. Then we discuss a priori and a posteriori strategies for choosing these parameters. Finally, we give numerical examples illustrating the theoretical results.
Zeros of random polynomials
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We consider global distribution of zeros for several ensembles of random polynomials. The main directions are related to almost sure limits of the zero counting measures, and to quantitative results on the expected number of zeros in various sets. In the simplest case of Kac polynomials given by the linear combinations of monomials with independent random coefficients, it is well known that their zeros are asymptotically uniformly distributed near the unit circumference under mild assumptions on the coefficients. We give estimates of the expected discrepancy between the zero counting measure and the normalized arclength on the unit circle. Similar results are established for polynomials with random coefficients spanned by different bases, e.g., by orthogonal polynomials. We show almost sure convergence of the zero counting measures to the corresponding equilibrium measures for associated sets in the plane, and quantify this convergence.

Asymptotic behavior of the partial derivatives of Laguerre kernels and some applications
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We will establish some new results corresponding to the asymptotic behavior of the partial derivatives of Laguerre kernels. Also we will show how these results can be used in order to obtain the inner relative asymptotics for certain Laguerre-Sobolev type polynomials.

Measurable diagonalization of positive definite matrices and applications to non-diagonal Sobolev orthogonal polynomials
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In this work we show that any positive definite matrix \( V \) with measurable entries can be written as \( V = U \Lambda U^* \), where the matrix \( \Lambda \) is diagonal, the matrix \( U \) is unitary, and the entries of \( U \) and \( \Lambda \) are measurable functions (\( U^* \) denotes the transpose conjugate of \( U \)). This property allows to obtain results about
the zero location and asymptotic behavior of extremal polynomials with respect to a generalized non-diagonal Sobolev norm in which products of derivatives of different order appear. The orthogonal polynomials with respect to this Sobolev norm are a particular case of those extremal polynomials.

References


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Constructive approximation of complex order modified Bessel functions

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The new realization of the Lanczos Tau method with minimal residue is proposed for the constructive approximation of the second order differential equations solutions with polynomial coefficients. The approximating scheme of Tau method is extended for the systems of hypergeometric type differential equations. A Tau method computational scheme is applied for the constructive approximation of a system of differential equations solutions related to the differential equation of hypergeometric type. Various vector perturbations are discussed. Our choice of the perturbation term is a shifted Chebyshev polynomial with a special form of selected transition and normalization. The minimality conditions for the perturbation term are found for one equation [1]. They are sufficient simple for the verification in a number of important cases. The constructive approximation of kernels of LEBEDEV type index transforms – modified Bessel functions of the second kind with complex order \( K_{\alpha+i\beta}(x) \) is elaborated in detail. The problem is reduced to the approximate solution of the system of Volterra integral equations by means of Chebyshev polynomial approximations. The new applications of LEBEDEV type integral transforms and related dual integral equations [2] for the numerical solution of some problems of mathematical physics are given. The algorithm of numerical solution of some mixed boundary value problems for the Helmholtz equation in wedge domains is developed. Observed examples admitting complete analytical solution demonstrate the efficiency of this approach for applied problems.
References


Polynomial Interpolation around the Bernstein Constant
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Let $\alpha > 0$ be not an even integer. In 1913 and later in 1938, S.N. Bernstein established the limit relation

$$B_{\alpha,\infty} = \lim_{n \to \infty} n^\alpha E_n (|x|^\alpha, L_\infty [-1,1]).$$

Here, $E_n (f, L_p [a,b])$ denotes the error in best $L_p$ approximation of a function $f$ on the interval $[a,b]$ by polynomials of maximal degree $n \in \mathbb{N}$. The constants $B_{\alpha,\infty}$ are the so-called Bernstein constants. There is not a single value of $\alpha$ for which $B_{\alpha,\infty}$ is explicitly known.

In this talk we present new limit relations for polynomial interpolations for $|x|^\alpha$, $\alpha > 0$. In addition, we discuss a construction of (near) best approximating polynomials for $|x|^\alpha$ in the $L_\infty$ norm, possibly giving a way towards a representation for the Bernstein constant.

Parameterized Orthogonal Wavelets
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When constructing wavelets, the researcher must choose from a variety of properties they wish to impose on the wavelet such as compact support, orthogonality, approximation order, etc. The approximation order is directly related to the number of vanishing moments of the wavelet. Daubechies sought orthogonal wavelets with a maximum number of vanishing moments for a given support length. In this talk, we will present a parameterization that includes all the wavelets of support length ten that includes the Daubechies wavelets among the continuum. By not imposing the multiple vanishing moments condition, the degrees of freedom can be used to construct orthogonal wavelets which perform better than the standard Daubechies wavelets in an image compression.
scheme by achieving a steeper transition in the frequency domain and achieve compression results comparable with the FBI biorthogonal 9/7 wavelet.

A glimpse of the history and properties of the Plastic Number
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In a paper from 2012 Brauchart, Dragnev, Saff and van de Woestijne studied what happens when a positive unit point charge is approaching the unit sphere \( S^d \), seen as a perfectly isolated spherical conductor with a total charge of +1, from infinity. The question asked was, what is the smallest distance, \( r(d) \) from the point charge to the sphere, so that the positive charge of the sphere covers all of the sphere. It turns out that this distance is closely connected to a root of a polynomial equation of degree \( 2d - 1 \). Furthermore, it turns out that \( r(2) \) is the Golden Ratio, and \( r(4) \) is the Plastic Number.

The Golden Ratio and its properties are certainly very well known but what about the Plastic Number? Searching into the history of the Plastic Number one quickly is led to a Dutch monk and architect, Dom Hans van der Laan. It also turns out that the Golden Ratio and the Plastic Number share a certain property, which they indeed are the only numbers to possess. This again links to some number theoretic results published by two Norwegian mathematicians more than 50 years ago.

In this very elementary talk I will look briefly into both the history and the properties of the Plastic Number.

Searching for optimal factorizations of rational numbers
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The metric Mahler measure, first explored by Dubickas and Smyth in 2001, identifies a way of writing an algebraic number \( \alpha \) as a product of algebraic numbers having various extremal properties. Nevertheless, the points which form this product are difficult to determine even in the special case where \( \alpha \in \mathbb{Q} \). We describe a method for locating these points using a certain type of tree data structure called a factorization tree, which we believe is of independent interest.
Sets of Minimal Logarithmic Capacity and Inverse Polynomial Images
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Consider a function $f$ analytic in a neighborhood of infinity. We want to find an extremal domain $D_0$ to which the function $f$ can be extended in an analytic and single-valued manner. The domain $D_0$ is extremal in the way that the complement of $D_0$, denoted by $K_0$, has minimal logarithmic capacity. For any polynomial $P$, we show that the inverse image $P^{-1}([-1,1])$ is an extremal domain $K_0$ for a certain function $f$. Note that the set $K_0$ is also crucial for the convergence theory of the Padé approximation of $f$.

New representation theorems for completely monotone and Bernstein functions with convexity properties on their measures
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We investigate a class of Bernstein and a class of completely monotone functions with intriguing applications in convex analysis. We derive representation theorems for Bernstein and completely monotone functions with a convexity condition on their measures. These representation theorems are variants of the classical Bernstein and Lévy-Khintchine representation theorems. We show that the transformations that turn a Bernstein function into one having corresponding Lévy measure with harmonically concave tail are the same as the transformations that transform a completely monotone function into one having harmonically convex measure.

Multiple orthogonal polynomials, a max-min energy problem ansatz and global trajectories of a quadratic differential
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Consider the sequence of multiple orthogonal polynomials \((P_{n,m})\), \(P_{n,m}(z) = z^{n+m} + \cdots\), defined by

\[
\int_{\Gamma_1} P_{n,m}(z) z^j e^{-Nz^3} dz = 0, \quad j = 0, \ldots, n - 1,
\]

\[
\int_{\Gamma_2} P_{n,m}(z) z^j e^{-Nz^3} dz = 0, \quad j = 0, \ldots, m - 1,
\]

where \(N = n + m\) and \(\Gamma_1 = [\infty e^{2\pi i/3}, 0] \cup [0, \infty], \Gamma_2 = [\infty e^{4\pi i/3}, 0] \cup [0, \infty e^{2\pi i/3}]\).

A classical problem is to describe the limiting zero distribution of \(P_{n,m}\), namely to characterize the weak limit

\[
\mu_0 = \lim_{N \to +\infty} \frac{1}{N} \sum_{P_{n,m}(w) = 0} \delta_w.
\]

Van Assche, Filipuk and Zhang recently studied the diagonal polynomials \((P_{n,n})\). Among other results, they computed explicitly an algebraic equation satisfied by the Cauchy transform of \(\mu_0\).

In this talk, we investigate the zeros of \(P_{n,m}\) in the non-diagonal limit

\[
\frac{n}{N} N \to \alpha, \quad 0 < \alpha < \frac{1}{2}.
\]

We propose a max-min vector equilibrium problem that should be solved by the measure \(\mu_0\). Although this ansatz is not rigorous - and indeed just correct for \(\alpha\) smaller than a certain critical value \(\alpha_c < \frac{1}{2}\) - it leads to an algebraic equation for the Cauchy transform of \(\mu_0\), valid for any \(\alpha\).

Motivated by the Riemann-Hilbert approach for asymptotics of orthogonal polynomials, we study the global behavior of trajectories of a certain quadratic differential on the Riemann surface defined by this algebraic equation.

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**Periodic Discrete Energy in Euclidean Space**

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Coauthors: D. P. Hardin and E. B. Saff

We will discuss energy problems associated to discrete periodic sets in Euclidean space. A special emphasis will be placed on long-range potentials that require a renormalization procedure to make sense of the energy. Such long-range potentials include Riesz potentials for small values of the Riesz parameter and the logarithmic potential. We will also discuss several properties of minimal energy configurations such as the leading order growth of the minimal energy and the macroscopic distribution of points in an energy minimizing configuration as the number of points becomes large.
Formulas and identities for basic hypergeometric series involving the
Askey-Wilson operator
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We use a certain representation of the iterated Askey-Wilson operator (a
variant of Cooper’s formula) to derive a number of summation and transforma-
tion formulas for basic hypergeometric series. We also establish a new integra-
tion by parts rule for the iterated Askey-Wilson operator and use it to derive
connection relations involving the Askey-Wilson polynomials. A generalized vari-
ant of the Leibnitz rule for the iterated Askey-Wilson operator is established.

Orthogonal Polynomials on Finite Gap Sets
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After reviewing the analogs of Szegő’s Theorem for $[-2, 2]$, we describe the
finite gap setup. The main focus will be to describe the Fuchsian group approach
and underlying mathematical framework although I’ll end with statements of
the Szegő Theorem analogs in this situation.

Solvable ensembles of random polynomials
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Coauthors: Maxim Yattselev

A height is a measure of complexity of polynomials. For certain multi-
plicative heights, choosing a polynomial of fixed degree uniformly from a set
of bounded height produces a point process on the roots similar to those that
arise in random matrix theory. I will talk generally about such processes, and
give more detailed information for specific heights (including Mahler measure).
Much of this follows from ongoing work with Maxim Yattselev.
Zooming in – Multiscale RBF approximation can be locally refined
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Coauthors: Q. Thong Le Gia and Holger Wendland

Physical phenomena on the earth’s surface occur on many different length scales, so it makes sense when seeking an efficient approximation to start with a crude approximation, and then make a sequence of corrections on finer and finer scales. It also makes sense for fine-scale approximations to be computed locally, rather than through a large global computation. In the present talk, describing recent joint work with Q. Thong Le Gia and Holger Wendland, we start with our global multiscale radial basis function (RBF) approximation scheme (SIAM J. Numer. Anal. 2010), based on a sequence of point sets with decreasing mesh norm, and a sequence of associated (spherical) radial basis functions with proportionally decreasing scale. We then prove that we can “zoom in” on a region of particular interest, by carrying out further stages of multiscale refinement on a local region. The process can be continued indefinitely, since the condition numbers of the matrices for different scales remain bounded. Colorful numerical experiments illustrate the possibilities.

Spectral Properties of Discrete Random Schrödinger Operators with Small Coupling Constants
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We consider one-dimensional discrete Schrödinger operators $H = \Delta + \lambda V$ with random potentials $V$ and coupling constants $\lambda > 0$. We describe the behavior of the density of states and the transition in the microscopic eigenvalue statistics of these operators, as the coupling constant $\lambda$ approaches 0.
Orthogonal Polynomials in the Complex Plane and Applications in Inverse Moment Problems
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Let $\mu$ be a finite positive Borel measure with compact support in the complex plane, and let $\{p_n(\mu, z)\}_{n=0}^{\infty}$ denote the sequence of the orthonormal polynomials, with positive leading coefficients, defined by the inner product

$$\langle f, g \rangle_\mu := \int f(z) \overline{g(z)} d\mu(z).$$

The purpose of the talk is to report on some recent developments regarding the asymptotics of $\{p_n(\mu, z)\}_{n=0}^{\infty}$, in cases when $\mu$ belongs to a special class of measures that includes area-type measures. This leads to algorithms for recovering the shape of the support of $\mu$, from a finite set of the associated moments $\langle z^i, z^j \rangle_\mu$, $i, j = 0, 1, \ldots, n$, and in particular, to applications in 2D geometric tomography.

On approximation properties of Shannon Sampling operators with bandlimited kernels
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The generalized sampling operator is given by ($t \in \mathbb{R}; w > 0$)

$$(S_w f)(t) := \sum_{k=-\infty}^{\infty} f \left( \frac{k}{w} \right) s(wt - k). \quad (5)$$

In this talk we study an even band-limited kernel $s$, defined as Fourier cosine transform of an even window function $\lambda \in C_{[-1, 1]}$, $\lambda(0) = 1$, $\lambda(u) = 0$ ($|u| \geq 1$).

We will estimate the order of approximation of the sampling operator (5) for functions $f$ belonging in a suitable subspace $\Lambda^p \subset L^p(\mathbb{R})$ (see also [1]) in terms of modulus of smoothness.

**Theorem 1** Let sampling operator $S_w^r$ ($w > 0$) be defined by the kernel $s$ with $\lambda = \lambda_r$ and for some $r \in \mathbb{N}$ let

$$\lambda_r(u) := 1 - \sum_{j=r}^{\infty} c_j u^{2j}, \quad \sum_{j=r}^{\infty} |c_j| \leq \infty. \quad (6)$$
Then for $f \in \Lambda^p$ ($1 \leq p < \infty$)

\[ \|S_r^w f - f\|_p \leq M_r \omega_{2r} \left( f; \frac{1}{w} \right)_p. \]  

(7)

The constants $M_r$ are independent of $f$ and $w$.

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The tale of a formula
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The formula in question is

\[ \mu_n = \cos \left( \frac{\pi}{2(n + 1)} \right)^{-n-1}, \]

and the talk will discuss its origin and its connections to some extremal problems on polynomials, to discrepancy theorems, to some problems of Erdős and to a conjecture of Widom. The main theme is how small a polynomial can get on a compact set if its main coefficient or its value at a given point is given.

Approximation theory seen through the eyes of Chebfun
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Half of what I know of approximation theory, I learned from Ed Saff. The rest has come from Chebfun. This talk will be an online exploration of some favorite approximation topics including Chebyshev series, best approximation, Padé, Chebyshev-Padé, Carathéodory-Fejér, Jentzsch and Walsh theorems, rational interpolation and least-squares, Lebesgue constants, the Runge phenomenon, Bernstein ellipses, and analytic continuation.
Asymptotics of hypergeometric orthogonal polynomials via difference equations
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In this talk, we review some recent developments on asymptotic analysis of hypergeometric orthogonal polynomials via difference equations. Analogue to the steepest-descent method for asymptotics of integrals and WKB approximations for asymptotics of differential equations, we propose a systematic technique for asymptotics of difference equations. By applying this new technique, we derive asymptotic formulas of Racah polynomials as the polynomial degree tends to infinity.

Computation of isotropic random fields on spheres via needlets decomposition
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Coauthors: Quoc Thong Le Gia (University of New South Wales)

Isotropic random fields on the sphere have applications in environmental models and astrophysics. The classic Karhunen-Loève expansion in terms of spherical harmonics has a drawback of requesting full observation of the random fields over the sphere. Attempting to solve this difficulty, we study the decomposition by needlets — a highly localised basis — of an isotropic random field on the sphere. We prove the $L_2$ convergence of the needlets decomposition of a two-weakly isotropic random field. We use a quadrature rule to construct fully discrete needlets for computational purpose and give the truncation error of the discrete needlets approximation for smooth isotropic random fields.

Kernel-Based Quadrature on Spheres and Other Manifolds, Part I
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This talk will present coordinate independent quadrature (or cubature) formulas associated with with certain positive definite and conditionally positive definite kernels that are invariant under the group action of the manifold. We will discuss the accuracy and stability of such formulas with special emphasis on spheres. Part II of this talk, presented by F. J. Narcowich, will present applications of these quadrature methods to meshless methods.
On a large deviation principle in potential theory
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Coauthors: Thomas Bloom and Norman Levenberg

Weighted potential theory is useful in large deviation results for the empirical measures of certain random matrix ensembles. In this talk, we will define and describe the main properties of a $L^2$–type discretization of weighted logarithmic energy with respect to a given measure $\nu$, in a scalar or vector setting. This leads to a large deviation principle for a canonical sequence of probability measures if $\nu$ satisfies some Bernstein-Markov property.

This is a joint work with Thomas Bloom and Norman Levenberg.

An open conjecture on the farthest distance function
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Let $E$ be any bounded set in $\mathbb{R}^N$. Then the farthest distance function is given by

$$d_E(x) = \sup_{y \in E} |x - y|.$$  

The farthest distance function can be expressed via the potential of a unique unit measure $\sigma_E$. This measure reflects the geometry of the set and has interesting connections to computational geometry and optimization theory.

Laugesen and Pritsker conjectured that $\sigma_E(E)$ is bounded above by $2^{1-N}$. This conjecture was proven in the planar case $N = 2$ by Gardiner and Netuka. There has also been recent work on some special cases for $N \geq 3$. We present the current state of work on the conjecture along with a variety of examples.

Minimax problems for choosing point sets on the sphere
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This talk considers both finite and continuous minimax problems arising from distributing points on the sphere. Best known are are point sets that maximize the minimum distance between distinct pairs of points (best packing).

On the other hand are point sets that minimize the maximum distance from a point on the sphere to a point in the set (best covering). Classically the interest has been in the asymptotic behaviour at the number of points increases. A more recent related problem, highlighted by Erdélyi and Saff, is that of maximizing
the polarization of the point set. Characteristics of solutions and algorithms for finding local solutions are considered. As usual for optimization problems on the sphere, there are difficulties with many local solutions.

**Asymptotics of the Meijer G-functions**

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Asymptotic expansions of the Meijer G-function are derived for large values of the variable. The derivation is simple and straightforward; it makes use of only the Cauchy residue theorem.

**IMS data analysis using Multi-resolution method and Markov Random Field**

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Coauthors: Don Hong

In this talk, I will present applications of wavelets in imaging mass spectrometry (IMS) data analysis for biomarker selection and classification. In addition to applying wavelets for data de-noising, the idea of wavelet pyramid method for image matching is applied for biomarker selection. Also, the Naive Bayes classifier was used for classification based on feature vectors in terms of wavelet coefficients. Performance of the algorithm was evaluated in real data applications. Experimental results show that this multi-resolution method has advantages of fast computing and robustness. To better incorporate spatial information, we further apply Markov Random Field (MRF) and Bayesian methods in IMS data analysis. Markov chain Monte Carlo (MCMC) is used to for computing implementation. Maximum pseudo likelihood method is used for parameter estimation.
Asymptotic expansions and sharp bounds for the Landau constants via a difference equation approach
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Coauthors: Yutian Li, Saiyu Liu and Yuqiu Zhao

In this talk, we study the asymptotic expansions and sharp bounds for the Landau constants $G_n$, which satisfies a second order linear difference equation. By applying the theory of Wong and Li for difference equations, we obtain the complete asymptotic expansion for the Landau constants

$$\pi G_n \sim \ln N + \gamma + 4 \ln 2 + \sum_{s=1}^{\infty} \beta_{2s}/N^{2s}, \quad n \to \infty,$$

where $N = n + 3/4$, $\gamma = 0.5772\cdots$ is Euler’s constant, and $(-1)^{s+1}\beta_{2s}$ are positive rational numbers, given explicitly in an iterative manner. Then we show that the error due to truncation is bounded in absolute value by, and of the same sign as, the first neglected term for all nonnegative $n$. Consequently, we obtain optimal sharp bounds for the Landau constants up to arbitrary orders of the form

$$\ln N + \gamma + 4 \ln 2 + \sum_{s=1}^{2m} \beta_{2s}/N^{2s} < \pi G_n < \ln N + \gamma + 4 \ln 2 + \sum_{s=1}^{2k-1} \beta_{2s}/N^{2s}$$

for all $n = 0, 1, 2, \cdots$, $m = 1, 2, \cdots$, and $k = 1, 2, \cdots$.

This is a joint work with Yutian Li, Saiyu Liu and Yuqiu Zhao.

Positive definite functions on the unit sphere
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Coauthors: R.K. Beatson and W. zu Castell

A simple sufficient condition, called Pólya criterion, is stated for a function $f(\cos \psi)$ to be positive definite on the unit sphere. The criterion is proved for lower dimension spheres and is conjectured for spheres of all dimensions. The essential ingredient is to show that, for $t \in (0, \pi)$, the function $(t-\psi)^r$ is positive definite on the sphere $S^{d-1}$ if $r \geq d/2$.  

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Meromorphic extendibility and rigidity of interpolation
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Let \( T \) be the unit circle, \( f \) be an \( \alpha \)-Hölder continuous function on \( T \), \( \alpha > 1/2 \), and \( \mathcal{A} \) be the algebra of continuous function in the closed unit disk \( \mathbb{D} \) that are holomorphic in \( \mathbb{D} \). Then \( f \) extends to a meromorphic function in \( \mathbb{D} \) with at most \( m \) poles if and only if the winding number of \( f + h \) on \( T \) is bigger or equal to \( -m \) for any \( h \in \mathcal{A} \) such that \( f + h \neq 0 \) on \( T \).

Strong Asymptotics of Hermite-Padé Approximants for Angelesco Systems with Complex Weights
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An Angelesco system with complex weights is a vector of Cauchy transforms of complex measures compactly and disjointly supported on the real line. Hermite-Padé approximant to such a system is a vector of rational functions all having the same denominator. I will discuss asymptotic properties of such approximants when the measures have smooth derivatives with respect to the Lebesgue measure.

Multiple orthogonal polynomials associated with an exponential cubic weight
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Coauthors: Walter Van Assche and Galina Filipuk

We consider multiple orthogonal polynomials associated with the exponential cubic weight \( e^{-x^3} \) over two contours in the complex plane. We study the basic properties of these polynomials, including the Rodrigues formula and nearest-neighbor recurrence relations. It turns out that the recurrence coefficients are related to a discrete Painlevé equation. The asymptotics of the recurrence coefficients, the ratio of the diagonal multiple orthogonal polynomials and the (scaled) zeros of these polynomials are also investigated. Joint work with Walter Van Assche and Galina Filipuk.
CMV Matrices with Super Exponentially Decaying Verblunsky Coefficients
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For the class of five diagonal unitary CMV matrices we establish bijective correspondences between the matrices with super exponentially decaying Verblunsky coefficients and spectral data associated with the Jost solution.

Minimum Riesz energy problems for a condenser with “touching plates”
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Coauthors: P. D. Dragnev, D. Hardin, E. B. Saff

We consider minimum $\alpha$-Riesz energy problems in the presence of an external field for a condenser with “touching plates” $A_1$ and $A_2$ in $\mathbb{R}^n$. An intimate relationship between such problems and minimum $\mathcal{G}_{\mathbb{R}^n \setminus A_2}^\alpha$ Green problems for positive measures on $A_1$ is discovered. We obtain sufficient and/or necessary conditions for the existence of minimizing measures for both the unconstrained and the constrained problems, and show their uniqueness. Furthermore, characterization theorems in terms of variational inequalities for the weighted potentials are established. The results are illustrated by some examples.