Title: Constructing new bases from old.

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Abstract: It is well know that every function in $L^2(0,1)$ can be represented as a Fourier series. The sequence $\{e^{2\pi inx}\}_{n\in\mathbf{Z}}$ or more generally, sequences $\{e^{2\pi i(n+d)x}\}\}_{n\in\mathbf{Z}}$, where d is a constant, is called a Fourier basis of $L^2(0,1)$.

Let I be a finite union of disjoint intervals of unit length. We construct a Fourier basis of $L^2(I)$ which is a union of Fourier bases of the individual intervals.

We also consider the following problem: In $\mathbf{R}^n$, (and in general in a Hilbert space) every element can be represented as a linear combination of its projections on an orthonormal basis $\{e_j\}$, and we have the Parseval identity $||f||^2 = \sum_{j=0}^{\infty} | < f, e_j > |^2$. If the basis is not orthonormal, the Parseval identity is replaced by the frame inequality

$$A\sum_{j=0}^{\infty} | < f, e_j > |^2 \leq ||f||^2 \leq B\sum_{j=0}^{\infty} | < f, e_j > |^2$$

where $) < A \leq B$. We ask about the best constants A and B in the inequality above when one of more element of the orthonormal basis $\{e_j\}$ is replaced by unit vectors which are not necessarily orthogonal. We give an exact answer in some special case.

Part of this work is joint with J. Edward (FIU) and Goheen Shaikh Samad (formerly a graduate student at FIU, now at the U. Iowa)