Title: Constructing new bases from old.
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Abstract: It is well known that every function in $L^2(0,1)$ can be represented as a Fourier series. The sequence $\{e^{2\pi inx}\}_{n \in \mathbb{Z}}$ or more generally, sequences $\{e^{2\pi i(n+d)x}\}_{n \in \mathbb{Z}}$, where $d$ is a constant, is called a Fourier basis of $L^2(0,1)$.

Let $I$ be a finite union of disjoint intervals of unit length. We construct a Fourier basis of $L^2(I)$ which is a union of Fourier bases of the individual intervals.

We also consider the following problem: In $\mathbb{R}^n$, (and in general in a Hilbert space) every element can be represented as a linear combination of its projections on an orthonormal basis $\{e_j\}$, and we have the Parseval identity $||f||^2 = \sum_{j=0}^{\infty} | \langle f, e_j \rangle |^2$. If the basis is not orthonormal, the Parseval identity is replaced by the frame inequality

$$A \sum_{j=0}^{\infty} | \langle f, e_j \rangle |^2 \leq ||f||^2 \leq B \sum_{j=0}^{\infty} | \langle f, e_j \rangle |^2$$

where $A < A \leq B$. We ask about the best constants $A$ and $B$ in the inequality above when one of more element of the orthonormal basis $\{e_j\}$ is replaced by unit vectors which are not necessarily orthogonal. We give an exact answer in some special case.

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